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Chapter 12

LINEAR PROGRAMMING

TERMINOLOGIES RELATED TO L.P.P:

Term	Definition
Objective function	The Linear function which is to be maximised or minimised under given constraints.
Constraints	The restrictions (or) Linear inequalities (or) equations in the variables of an L.P.P, which describe the conditions under which the optimisation is to be accomplished.
Non- negativity constraints	The assumption that negative values of variables are not possible in the solution. They are described as $x \geq 0, y \geq 0$.
Feasible solution	The set of points , whose co-ordinates satisfy the constraints and non-negativity of a linear programming problem, is said to be the feasible solution.
Infeasible solution	A solution of L.P.P is an infeasible solution, if it does not satisfy the non-negativity restrictions.
Optimal feasible solution	A feasible solution of L.P.P which maximizes (or) minimizes the objective function is called its Optimal solution.
Optimal value of objective function	The Optimal value (maximum or minimum of) the objective function is obtained at a vertex of the feasible region(if it is bounded). If there is more than one point (vertex) where the objective function is optimum (maximum or minimum), then every point on the line segment joining any two such vertices optimizes the objective function.
Feasible region	The common region determined by all the constraints including non-negative constraints of a L.P.P is called the feasible region (or) solution region. Feasible region is always a convex set. Feasible region may be bounded (or) unbounded.
Infeasible region	The region other than the feasible region is called Infeasible region of the L.P.P.
Corner point of a feasible region	A point in the feasible region which is the intersection of two boundary lines is called Corner point.
Infinite number of Optimal solutions	The L.P.P may have more than one solution because of the nature of the objective function. Each of such optimal solutions is referred to alternative optimal solution. Hence , there can be infinite number of Optimal solutions.

DIFFERENT TYPES OF L.P.P:

Term	Definition
Diet problems	In diet problems, amount of different kinds of nutrients/constituents which are to be included in a diet are determined so as to minimize the cost of the desired diet such that it contains a certain minimum amount of nutrient or constituent.
Manufacturing problems	In these problems, we determine the number of units of different production which should be produced and sold by a firm when each product requires a fixed man power, machine hours, labour hour per unit of product, warehouse space per unit of the output etc. In order to make maximum profit.
Transportation	In such type of problems a transportation schedule, so as to find the cheapest way of transporting a product from plants/factories situated at different locations to different markets is to be determined so as to minimize the cost of transportation subject to limitations (constraints) of demand of each market and supply from each plant (or) factory.
Investment problems	Investment portfolio from a variety of stocks or bonds is established that will maximize company's return on investments
Advertising Firms problems	Effectiveness of advertising activities in the field like T.V, radio, newspapers is maximized by allocation of limited advertising budget

FORMULATION OF A L.P.P:

The three steps in the mathematical formulation of an L.P.P are as follows:

(i) To identify the objective function as a linear combination of variables (x and y) and to construct all constraints. i.e. Linear equations and inequations involving these variable. Thus, an L.P.P can be stated mathematically as

Maximise (or Minimize) $z = ax + by$ subject to the constraints $a_i x + b_i y \le (\text{or } \ge \text{or } = \text{or} > \text{or} <) c_i$, where $i = 1$ to n $x \ge 0, y \ge 0$ (non-negative constraints).

(ii) To find the solutions (feasible region) of these equation and in equations by some mathematical method.

(iii) To find the optimal solution is to select particular values of the variables x and y that give the desired value (maximum/minimum) of the objective function.

SOME IMPORTANT THEOREMS OF L.P.P

Theorem-1:

Let R be the feasible region(convex polygon) for a linear programming problem and Let $z = ax + by$ be the objective function. When z has an optimal value (maximum or minimum), where the variables x and y are subject to constraints described by linear inequalities this optimal value must occur at a corner point(vertex) of the feasible region.

Theorem-2 :

Let R be the feasible region for a linear programming problem, and let $z = ax + by$ be the objective function. If R is bounded, then the objective function z has both a maximum and a minimum value on R and each of these occurs at a Corner point (vertex) of R.

Method To Solve Linear Programming Problem

Corner Point Method:

1. Find the feasible region of the linear programming problem and determine its Corner point (vertices).
2. Evaluate the objective function $z = ax + by$ at each corner point. Let M and m respectively denotes the largest and smallest values at these point.
3. When the feasible region is bounded, M and m are the maximum and minimum values of z.
4. In case, the feasible region is unbounded, then

(a) M is the maximum value z if open half plane determined by $ax + by > M$ has no point in common with the feasible region. Otherwise, z has no, maximum value.

(b) m is the minimum value of z if open half plane determined by $ax + by < M$ has no point in common with the feasible region. Otherwise , z has no minimum value.

NOTE:

If two corner points of the feasible region are both optimal solution of the same type i.e. both produce the same maximum (or) minimum, then any points is also an optimal solution of the same type.