Chapter 07

INDEFINITE INTEGRAL

METHODS OF INTEGRATION

Integration by substitution:

Direct Substitution:

- $$\int \left(f(x)\right)^n f'(x)dx = \frac{\left(f(x)\right)^{n+1}}{n+1} + c$$

  Let $f(x) = t \Rightarrow f'(x)dx = dt$

  $$\Rightarrow \int t^n \ dt = \frac{t^n + 1}{n+1} + c = \frac{\left(f(x)\right)^{n+1}}{n+1} + c$$

- $$\int \frac{f'(x)}{f(x)} dx = \log_e \left| f(x) \right| + c$$

  Let $f(x) = t \Rightarrow f'(x) \ dx = dt$

  $$\Rightarrow \int \frac{1}{t} \ dt = \log|t| + c = \log\left| f(x) \right| + c$$

- $$\int \frac{f'(x)}{f(x)} dx = 2\sqrt{f(x)} + c$$

  Let $f(x) = t \Rightarrow dt = f'(x)dx$

  $$\Rightarrow \int \frac{dt}{\sqrt{t}} = 2\sqrt{t} + c = 2\sqrt{f(x)} + c$$

- $\int f(ax+b)dx = \frac{1}{a}\int f(t)dt + c$, where $t = ax + b$

- $$\int \frac{f'(x)}{f(x)} dx = \log\left| f(x) \right| + c$$

- $\int [f(x)]^n f'(x)dx = \frac{[f(x)]^{n+1}}{n+1} + c$
- $\int f(x^n)x^{n-1}\,dx = \frac{1}{n}\int f(t)\,dt$, where $t = x^n$
- $\int F(f(x))f'(x)dx = \int F(t)dt + c$, where $f(x) = t$

EXAMPLE- 1

$$\int \frac{(1+x)e^x}{\cos^2(xe^x)}\,dx\ ,\ \cos(xe^x) \neq 0$$

SOLUTION:

Put,

$xe^x = t$

$\Rightarrow (xe^x + 1.e^x)dx = dt$

$\Rightarrow e^x(x+1)dx = dt$

$\therefore \int \frac{dt}{\cos^2 t} = \int \sec^2(t)\,dt = \tan x + C = \tan(xe^x) + C$.

EXAMPLE- 2:

$$\int \frac{1}{\sqrt{\sin^{-1} x}\,\sqrt{1-x^2}}\,dx\ ;\ x \in (0,1)$$

SOLUTION:

Put, $\sin^{-1} x = t$

Differentiate with respect to 'x',

$\frac{1}{\sqrt{1-x^2}}\,dx = dt$

$\Rightarrow \int \frac{1}{\sqrt{t}}\,dt$

$= 2\sqrt{t} + C$

$= 2\sqrt{\sin^{-1} x} + C$.