Brainbox learn easy

Chapter 07

INDEFINITE INTEGRAL

The Process of Differentiation and Integration are Inverses of each other in the sense that

1. $\frac{d}{dx}\left(\int f(x)dx\right)=\frac{d}{dx}\left(F(x)+C\right)=F'(x)=f(x)$

2. $\int\left[\frac{d}{dx}\left(f(x)\right)\right]dx=\int f'(x)dx=f(x)+C$

Where 'C' is an arbitrary real constant.

Example: $d\left(x^3\right)=3x^2dx$ and $\int 3x^2dx=x^3$

Infact $d\left(x^3+C\right)=3x^2dx\Rightarrow\int 3x^2dx=x^3+C.$

INDEFINITE INTEGRALS:

If $\frac{dF}{dx}=f(x)$, then $F(x)$ is called an indefinite integral (anti – derivative or primitive of $f(x)$).

$\int f(x)dx=F(x)+x=c$, where c is an arbitrary constant. The integral operator is linear.

- $\int\left(f(x)\pm g(x)\right)dx=\int f(x)dx\pm g(x)dx$
- $\int kf(x)dx=k\int f(x)dx, k$ is a constant.

IMPORTANT INTEGRALS:

1. $\int x^a dx = \frac{x^{a+1}}{a+1} + c, a \neq -1$

2. $\int \frac{1}{x} dx = 1n|x| + c$

3. $\int e^x dx = e^x + c$

4. $\int a^x dx = \frac{a^x}{1n\, a} + c, a > 0, a \neq 1$

5. $\int Sinx dx = -\cos x + c$

6. $\int \cos x dx = \sin x + c$

7. $\int \tan x\, dx = 1n|\sec x| + c$

8. $\int \cot x\, dx = 1n|\sin x| + c$

9. $\int \sec x\, dx = 1n|\sec x + \tan x| + c = 1n \tan\left(\frac{x}{2} + \frac{\pi}{4}\right) + c$

10. $\int \operatorname{cosec} x\, dx = 1n|\operatorname{cosec} x - \cot x| + c = 1n \tan \frac{x}{2} + c$

11. $\int \sec^2 x dx = \tan x + c$

12. $\int \operatorname{cosec}^2 x dx = -\cot x + c$

13. $\int \sec x \tan x dx = \sec x + c$

14. $\int \operatorname{cosec} x \cot x dx = -\operatorname{cosec} x + c$

15. $\int \frac{dx}{x^2+a^2} = \frac{1}{a}\tan^{-1}\frac{x}{a} + c$

16. $\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left|\frac{x-a}{x+a}\right| + c$

17. $\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\ln\left|\frac{a+x}{a-x}\right| + c$

18. $\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\frac{x}{a} + c$

19. $\int \frac{dx}{\sqrt{a^2-x^2}} = \ln\left|x+\sqrt{x^2-a^2}\right| + c$

20. $\int \frac{dx}{\sqrt{x^2+a^2}} = \ln\left|x+\sqrt{x^2+a^2}\right| + c$

21. $\int \frac{dx}{x\sqrt{x^2-a^2}} = \frac{1}{a}\sec^{-1}\frac{x}{a} + c$

22. $\int \sqrt{a^2-x^2}\,dx = \frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a} + c$

23. $\int \sqrt{a^2+x^2}\,dx = \frac{x}{2}\sqrt{a^2+x^2} + \frac{a^2}{2}\ln\left|x+\sqrt{x^2+a^2}\right| + c$

24. $\int \sqrt{x^2-a^2}\,dx = \frac{x}{2}\sqrt{x^2-a^2} - \frac{a^2}{2}\ln\left|x+\sqrt{x^2-a^2}\right| + c$