

Chapter **5**

COMPLEX NUMBERS AND QUADRATIC EQUATIONS

❖ Determine the sign of $x^2 - 5x + 6$ expression for $x \in R$

Sol: The roots of $x^2 - 5x + 6$ are 2 and 3

$\therefore$ If $2 < x < 3$, then $x^2 - 5x + 6$ coefficient of x^2 have opp. signs

If $x < 2$ or $x > 3$, then $x^2 - 5x + 6$, coefficient of x^2 have the same sign.

Quadratic in equation:

- If $ax^2 + bx + c$, a, b, c real, is a quadratic expression than $ax^2 + bx + c > 0$ or $ax^2 + bx + c \geq 0$ or is called quadratic in equation (or) inequality.

Eg: $x^2 - 7x + 12 > 0$, $2x^2 - 3x - 4 \leq 0$

Solving quadratic inequalities:

- In this method, finding solution by observing the sign changes, of quadratic expression.

Note: $ax^2 + bx + c \geq 0 \Rightarrow a(x - \alpha)(x - \beta) \geq 0$ ($\alpha < \beta$)

$x \in (\alpha, \beta) \Rightarrow \alpha < x < \beta$ for $a(x - \alpha)(x - \beta) < 0$

$$x \in (-\infty, \alpha) \cup (\beta, \infty) \Rightarrow x < \alpha \text{ or } x > \beta \text{ for}$$

$$a(x - \alpha)(x - \beta) > 0$$

Solve $x^2 + x - 12 \leq 0$.

Sol: $x^2 + 4x - 3x - 12 \leq 0$

$$x(x + 4) - 3(x + 4) \leq 0$$

$$(x + 4)(x - 3) \leq 0 \quad (\because a(x - \alpha)(x - \beta) \leq 0 \Rightarrow \alpha \leq x \leq \beta)$$

$$\Rightarrow -4 \leq x \leq 3 \text{ (or) } x \in [-4, 3]$$

2. Find the values of 'm' for which the equation have equal roots?

$$x^2 - 15 - m(2x - 8) = 0$$

Sol: $x^2 - 2mx + 8m - 15 = 0$ has equal roots

$$\Rightarrow b^2 - 4ac = 0 \Rightarrow (-2m)^2 - 4(8m - 15) = 0$$

$$\Rightarrow 4m^2 - 4(8m - 15) = 0 \text{ (cancel '4')}$$

$$\Rightarrow m^2 - 8m + 15 = 0$$

$$\Rightarrow (m - 3)(m - 5) = 0$$

$$\Rightarrow m = 3 \text{ or } 5$$

3. If $x^2 + 4ax + 3 = 0$ and $2x^2 + 3ax - 9 = 0$ have a common root, then find the values of a and the common root.

Sol: If α is the common root then

$$\alpha^2 + 4a\alpha + 3 = 0 \longrightarrow (1)$$

$$2\alpha^2 + 3a\alpha - 9 = 0 \longrightarrow (2)$$

$$2(1) - (2) \Rightarrow 5a\alpha + 15 = 0 \Rightarrow a\alpha = -3$$

$$(1) \Rightarrow \alpha^2 - 12 + 3 = 0 \Rightarrow \alpha^2 = 9 \Rightarrow \alpha = \pm 3$$

$$a\alpha = -3 \Rightarrow a(+3) = -3 \Rightarrow a = \pm 1$$

Question: Show that none of the values of the function $\frac{x^2 + 34x - 71}{x^2 + 2x - 7}$ over $\mathbb{R}$ lies between 5 and 9.

Sol: Let $y = \frac{x^2 + 34x - 71}{x^2 + 2x - 7}$

$$\Rightarrow yx^2 + 2xy - 7y = x^2 + 34x - 71$$

$$\Rightarrow (y-1)x^2 + (2y-34)x + (71-7y) = 0$$

$$x \in \mathbb{R}, b^2 - 4ac \geq 0 \text{ as } (2y-34)^2 - 4(y-1)(71-7y) \geq 0$$

$$\Rightarrow 4y^2 + 1156 - 136y - 4[71y - 7y^2 - 71 + 7y] \geq 0$$

$$\Rightarrow 4y^2 + 1156 - 136y - 284y + 28y^2 + 284 - 28y \geq 0$$

$$\Rightarrow 32y^2 - 448y + 1440 \geq 0$$

$$\Rightarrow y^2 - 14y + 45 \geq 0$$

$$\Rightarrow (y-5)(y-9) \geq 0 \left(\begin{array}{l} \because a(x-\alpha)(x-\beta) \geq 0 \\ x \in (-\infty, \alpha] \cup [\beta, \infty) \end{array} \right)$$

$$\Rightarrow y \in (-\infty, 5] \cup [9, \infty)$$

None of the values over $\mathbb{R}$ lies between 5 and 9

