

Chapter **5**

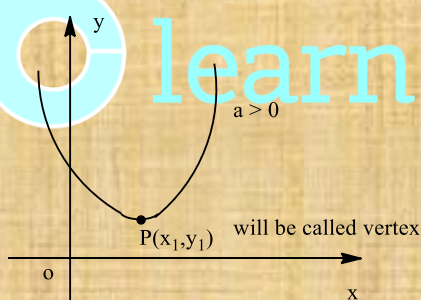
COMPLEX NUMBERS AND QUADRATIC EQUATIONS

Analysis of $y = ax^2 + bx + c$:

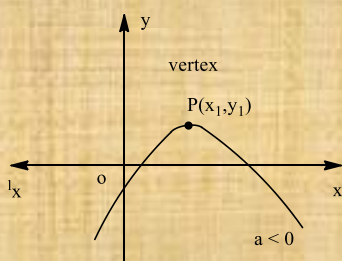
What do we mean by roots? Roots are the values of 'x' for which 'y' is equal to zero.

The graph of $y = ax^2 + bx + c$ (a quadratic polynomial) will always be a parabolic curve. (animation  or )

- If $a > 0$, the curve of parabola equation $y - y_1 = a(x - x_1)^2$ will be opening upward as follows.



- If $a < 0$, the curve of parabola equation $y - y_1 = a(x - x_1)^2$ will be opening downward as follows.



Also $y = ax^2 + bx + c$ is written in the form

$$y + \frac{(b^2 - 4ac)}{4a} = a \left(x + \frac{b}{2a} \right)^2$$

(or)

$$y + \frac{\Delta}{4a} = a \left(x + \frac{b}{2a} \right)^2 \longrightarrow (1)$$

$$\Delta = b^2 - 4ac$$

$$y - y_1 = a(x - x_1)^2 \longrightarrow (2)$$

Comparing (1) & (2)

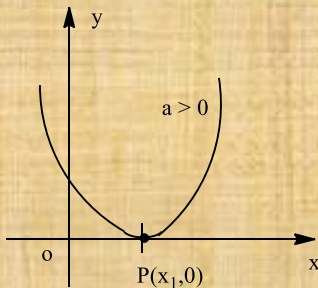
$$x_1 = \frac{-b}{2a}; y_1 = \frac{-\Delta}{4a}$$

$$\therefore \text{Vertex} = (x_1, y_1) = \left(\frac{-b}{2a}, \frac{-\Delta}{4a} \right).$$

Nature of Roots:

Suppose $a, b, c \in \mathbb{R}$ and $a \neq 0$

If $a > 0$ (Parabola will be upward).



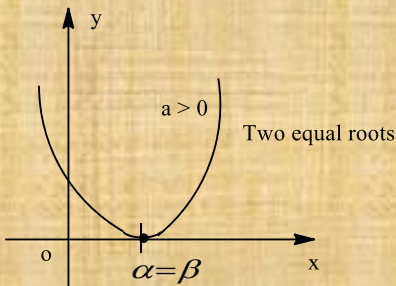
(i) $\Delta = 0$

- For this case, the vertex of the parabola.

$$y_1 = -\frac{\Delta}{4a} = 0 / \because \Delta = 0$$

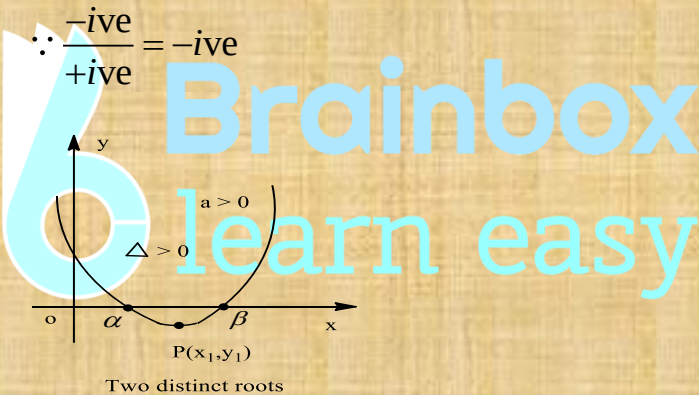
- The curve will just touch the x – axis only once.

- Thus for $\Delta = 0$, the quadratic equation will have only one root (or) both roots are coinciding.



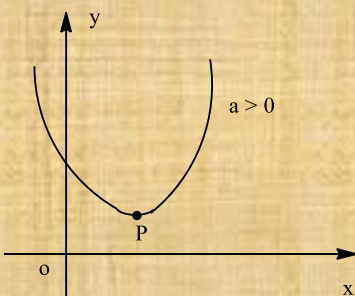
- (ii) $\Delta > 0$, think of y – coordinate of vertex of P of the parabola.

$$\text{y – coordinate of } P = -\frac{\Delta}{4a} = -\text{ive}$$



- The curve is being cut by x – axis at two places. Thus number of roots of this equation will be two. So, for real roots, $\Delta > 0$.

- (iii) Now $\Delta < 0$



- Think of y – coordinate of vertex P of a parabola;

$$y = -\frac{\Delta}{4a} = \frac{(-ive)(-ive)}{(+ive)}$$

$$y = +ive$$

- So, the curve will never touch the x – axis the quadratic equation will not have any real roots.
- So, the roots are imaginary for $\Delta < 0$.

