

Chapter **5**

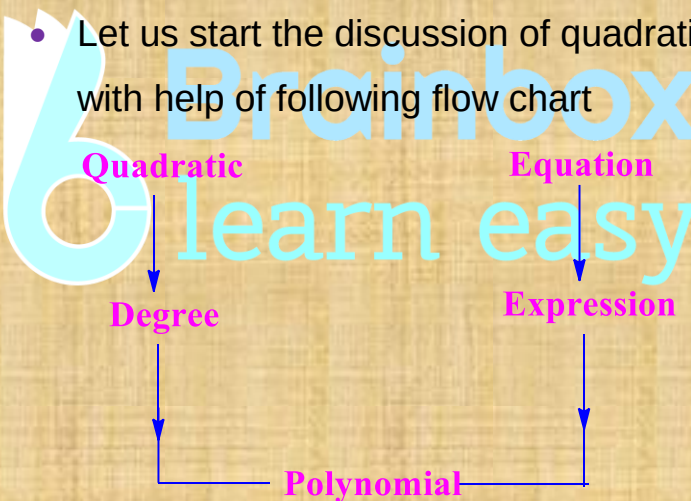
COMPLEX NUMBERS AND QUADRATIC EQUATIONS

Introduction:

Quadratic equations are used in many areas of science and engineering. The path of projectile animation of is almost parabolic and we use a quadratic equation to find out where the projectile is going to hit.

Quadratic equation:

- Let us start the discussion of quadratic equation with help of following flow chart



- There, before knowing what is quadratic equation, we have to understand the terms related to it, is expression, degree, polynomial etc.

Expression:

- It is defined as combinations of terms containing constant & variables.

a) Constants: Constants are the elements (entities) whose values is always fixed.

Generally, constants are denoted by a, b, c

b) Variables: Variables are those elements whose value changes under different cases/conditions, generally denoted by x, y, z.

Expression can be written as $ax^2 + bx$, $2x + 3$,

$9x^3 + bx^2 + 3x - 1$, $5\sin x + 2x$.

Algebra Equation:

When expression is equated to zero or one expression is equated to another expression than the result would be an equation.

Eg: $ax - bx^2 = 0$; $2x - 3 = 0$; $ax^2 + bx = cx^3 - d$

Roots of equation/zeros of equation:

- The values of variables, which when substitute in equation satisfy the equation are said to be zeros/ roots of equation.

Eg: $x^2 - 16 = 0$, when we put $x = 4$ or -4 then L.H.S = R.H.S, So $x = 4$ & -4 are the two roots of equation

$$x^2 - 16 = 0$$

Polynomial:

- It is an algebraic expression containing any number of terms (remember that number of terms in any of the polynomial must be finite)

Eg: $a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \dots + a_n$ is a polynomial where $a_0, a_1, a_2, \dots, a_n$ are constants and x is variable, n is a whole number.

Eg: $ax^3 + bx^2 + cx$

Degree of polynomial:

Highest power of variable 'x' in a polynomial is called a degree of polynomial.

Eg: $a_0 + a_1x + a_2x^2 + \dots + a_nx^n$ is n degree polynomial.

Quadratic expression:

A polynomial of degree two of the form $ax^2 + bx + c$ ($a \neq 0$) is called a quadratic expression in x .

Eg: $2x^2 + x + 1$

Quadratic expression:

A quadratic polynomial when equated to zero is called quadratic equation.

$$ax^2 + bx + c \quad (a \neq 0)$$

E.g. $x^2 - 5x + 6 = 0$

Roots (or) solutions of quadratic equation:

The values of variable x which satisfy the quadratic equation is called as roots (also called solutions or zeroes) of a quadratic equation.

Factorization method:

$$\text{Let } ax^2 + bx + c = a(x - \alpha)(x - \beta) = 0$$

Then $x = \alpha$ and $x = \beta$ will satisfy the equation. Hence factorize the equation and equating each to zero gives roots of equation.

Hindu method (Sridharacharya method):

Quadratic equation $ax^2 + bx + c = 0$ ($a \neq 0$) has two roots given by $\alpha = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$ and $\beta = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$

Here, the term $b^2 - 4ac$ is called discriminant of the equation it is denoted by Δ (delta) or D

Conjugate root:

The irrational and complex roots of a quadratic equation are always occurs in pairs.

Therefore ($a, b, c \in \mathbb{Q}$).

- If one root $\rightarrow \alpha + i\beta$ then another root $\rightarrow \alpha - i\beta$
- If one root $\rightarrow \alpha + \sqrt{\beta}$ then another root $\rightarrow \alpha - \sqrt{\beta}$

Some and product of roots:

If the roots of a quadratic equation $ax^2 + bx + c$
 $= (a \neq 0)$ are α and β then.

- Sum of roots;

$$S = \alpha + \beta = \frac{-b}{a} = \frac{-(\text{Co-efficient of } x)}{(\text{Co-efficient of } x^2)}$$

- And product of roots;

$$P = \alpha\beta = \frac{c}{a} = \frac{\text{constant term}}{\text{Co-efficient of } x^2}$$

Formation of an equation with given roots:

A quadratic equation whose roots are α and β is
 given by $x^2 - (\text{sum of roots})x + \text{Product of roots} = 0$ (or)

$$x^2 - (\alpha + \beta)x + \alpha\beta = 0$$

