

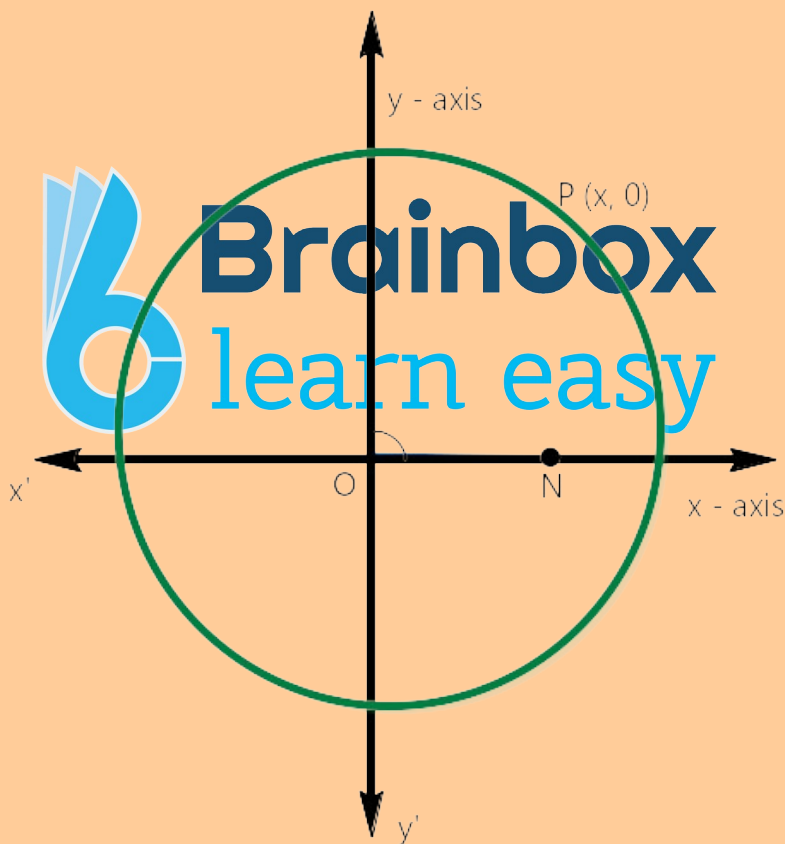
CHAPTER 08

Introduction to Trigonometry

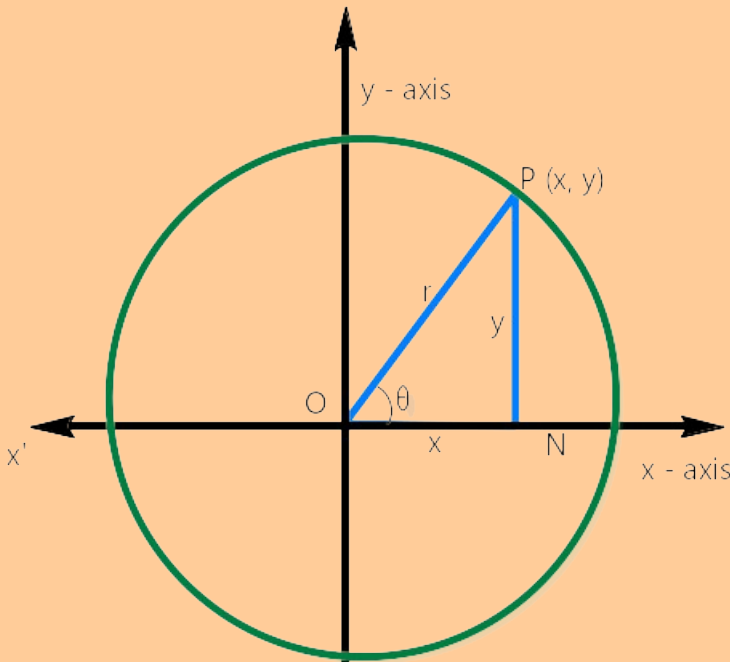
Trigonometric ratios of 90° :

Consider a coordinate system.

Let us increase the angle ' θ '. As the angle ' θ ' goes on increasing, the adjacent side (x) gradually decreases.



When ' θ ' becomes 90° , the adjacent side (x) vanishes and the hypotenuse OP coincides with the y – axis.



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So, when $\theta = 90^\circ$, adjacent side (Base) $(x) = 0$

Opposite side = Perpendicular side = $(y) = \text{Hypotenuse } (r) = r$

$$\sin 90^\circ = \frac{\text{Opposite}}{\text{Hypotenuse}} = \frac{y}{r} = \frac{y}{y} = 1$$

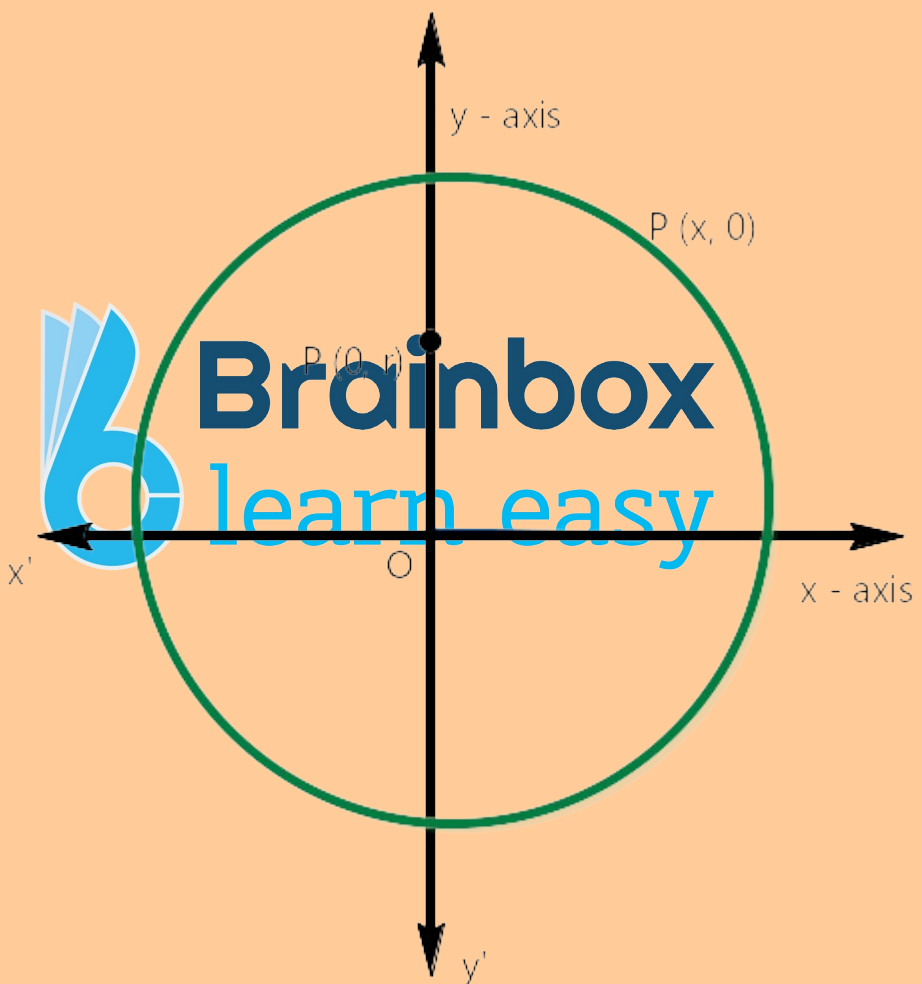
$$\Rightarrow \operatorname{Cosec} 90^\circ = \frac{\text{Hypotenuse}}{\text{Opposite}} = \frac{r}{y} = \frac{y}{y} = 1$$

$$\cos 90^\circ = \frac{\text{Adjacent}}{\text{Hypotenuse}} = \frac{x}{r} = \frac{0}{y} = 0$$

$$\Rightarrow \sec 90^\circ = \frac{\text{Hypotenuse}}{\text{Adjacent}} = \frac{y}{x} = \frac{y}{0} = \infty$$

$$\tan 90^\circ = \frac{\text{Opposite}}{\text{Adjacent}} = \frac{y}{x} = \frac{y}{0} = \infty$$

$$\Rightarrow \cot 90^\circ = \frac{\text{Adjacent}}{\text{Opposite}} = \frac{x}{y} = \frac{0}{y} = 0$$



Summary:

$$\sin 90^\circ = 1; \cos 90^\circ = 0; \tan 90^\circ = \text{Not defined}$$

$$\operatorname{cosec} 90^\circ = 1; \sec 90^\circ = \text{Not defined}; \cot 90^\circ = 0$$

Values of trigonometric functions of some standard angles:

T-ratios (θ)	$\sin \theta$	$\cos \theta$	$\tan \theta$	$\operatorname{cosec} \theta$	$\sec \theta$	$\cot \theta$
0°	0	1	0	Not defined	1	Not defined
30°	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$	2	$\frac{2}{\sqrt{3}}$	$\sqrt{3}$
45°	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	1	$\sqrt{2}$	$\sqrt{2}$	1
60°	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$	$\frac{2}{\sqrt{3}}$	2	$\frac{1}{\sqrt{3}}$
90°	1	0	Not defined	1	Not defined	0

Note:

- From the above table, it can be seen that the value of $\sin \theta$ increases from 0 to 1 and that of $\cos \theta$ decreases from 1 to 0 as θ increases from 0° to 90° .
- Since, the hypotenuse is the longest side in a right triangle, so the value of $\sin A$ or $\cos A$ is always less than 1 (Or in particular equal to 1), whereas the value of $\sec A$ or $\csc A$ is always greater than or equal to 1.

Example 1:

Evaluate

(i) $\frac{2 \tan 30^\circ}{1 + \tan^2 30^\circ}$

(ii) $\frac{1 - \tan^2 45^\circ}{1 + \tan^2 45^\circ}$

Sol.

(i)
$$\frac{2 \tan 30^\circ}{1 + \tan^2 30^\circ} = \frac{2 \times \left(\frac{1}{\sqrt{3}}\right)}{1 + \left(\frac{1}{\sqrt{3}}\right)^2} = \frac{\left(\frac{2}{\sqrt{3}}\right)}{1 + \left(\frac{1}{3}\right)} = \frac{\left(\frac{2}{\sqrt{3}}\right)}{\frac{3+1}{3}} = \left(\frac{2}{\sqrt{3}}\right) \times \frac{3}{4}$$

$$= \frac{3}{2} \times \frac{1}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{\cancel{3}\sqrt{3}}{2 \times \cancel{3}} = \frac{\sqrt{3}}{2}$$

$$(ii) \frac{1 - \tan^2 45^\circ}{1 + \tan^2 45^\circ} = \frac{1 - (1)^2}{1 + (1)^2} = \frac{1 - 1}{1 + 1} = \frac{0}{2} = 0$$

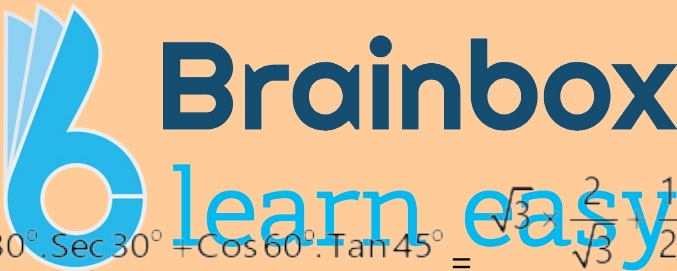
Example 2:

Evaluate

$$(i) \cot 30^\circ \cdot \sec 30^\circ + \cos 60^\circ \cdot \tan 45^\circ$$

$$(ii) \sin 60^\circ \cdot \cos 30^\circ + \cos 90^\circ \cdot \sin 60^\circ$$

Sol.



$$(i) \cot 30^\circ \cdot \sec 30^\circ + \cos 60^\circ \cdot \tan 45^\circ = \sqrt{3} \times \frac{2}{\sqrt{3}} + \frac{1}{2} \times 1 = 2 + \frac{1}{2}$$

$$= \frac{4 + 1}{2} = \frac{5}{2}$$

(ii)

$$\sin 60^\circ \cdot \cos 30^\circ + \cos 90^\circ \cdot \sin 60^\circ = \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} + 0 \times \frac{\sqrt{3}}{2} = \frac{3}{4} + 0 = \frac{3}{4}$$

Example 3:

By what number $\sin 45^\circ$ must be multiplied to get $\tan 60^\circ$.

Sol.

We know that,

$$\sin 45^\circ = \frac{1}{\sqrt{2}} \text{ and } \tan 60^\circ = \sqrt{3}$$

Let 'x' be the required number.

By given data,

$$x \times \sin 45^\circ = \tan 60^\circ$$

$$\Rightarrow x \times \frac{1}{\sqrt{2}} = \sqrt{3}$$

$$\Rightarrow x = \sqrt{3} \times \sqrt{2}$$

$$\therefore x = \sqrt{6} \quad \left(\because \sqrt{a} \times \sqrt{b} = \sqrt{ab} \right)$$

Example 4:

If $\sin(A - B) = \frac{1}{2}$ and $\cos(A+B) = \frac{1}{2}$, then find 'A' and 'B'.

Sol.

Given that,

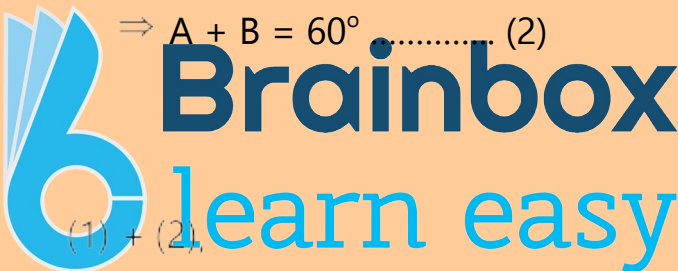
$$\sin(A - B) = \frac{1}{2} \Rightarrow \sin(A - B) = \frac{1}{2} = \sin 30^\circ$$

$$\Rightarrow A - B = 30^\circ \dots\dots\dots (1)$$

$$\cos(A + B) = \frac{1}{2} \Rightarrow \cos(A + B) = \frac{1}{2} = \cos 60^\circ$$

$$\Rightarrow A + B = 60^\circ \dots\dots\dots (2)$$

Now,



$$\begin{array}{r} A - B = 30^\circ \\ (+) \quad A + B = 60^\circ \\ \hline 2A = 90^\circ \\ \Rightarrow A = \frac{90^\circ}{2} \\ \Rightarrow A = 45^\circ \end{array}$$

By putting $A = 45^\circ$ in (1), we get

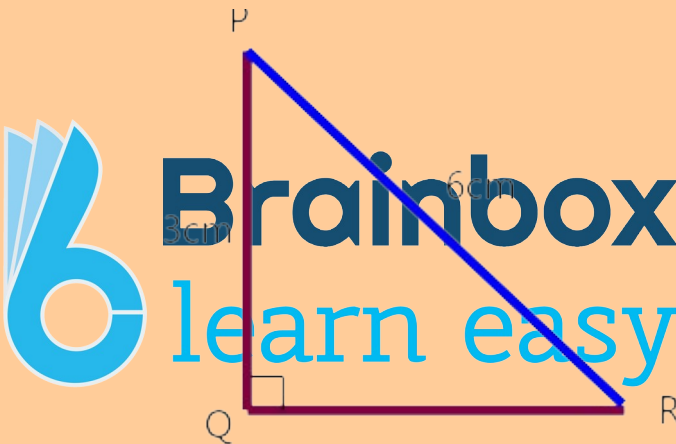
$$45^\circ - B = 30^\circ \Rightarrow B = 45^\circ - 30^\circ = 15^\circ$$

Hence,

$$A = 45^\circ \text{ and } B = 15^\circ$$

Example:

In the given figure, $\triangle PQR$ is right – angled at Q. $PQ = 3$ cm and $PR = 6$ cm. Determine $\angle QPR$ and $\angle PRQ$.



Sol.

Given,

$PQ = 3$ cm and $PR = 6$ cm

Clearly,

$$\sin R = \frac{\text{Opposite}}{\text{Hypotenuse}} = \frac{PQ}{PR} = \frac{3}{6} = \frac{1}{2}$$

$$\Rightarrow \sin R = \frac{1}{2}$$

$$\Rightarrow \sin R = \frac{1}{2} = \sin 30^\circ \quad \left(\because \sin 30^\circ = \frac{1}{2} \right)$$

$$\Rightarrow \boxed{\underline{R} = 30^\circ \text{ (or) } \underline{PRQ} = 30^\circ}$$

Now,

$$\text{In } \triangle PQR, \underline{P} + \underline{Q} + \underline{R} = 180^\circ$$

($\because$ By angle sum property of triangle)

$$\Rightarrow \underline{P} + 90^\circ + 30^\circ = 180^\circ$$

$$\Rightarrow \underline{P} = 180^\circ - 120^\circ$$

$$\Rightarrow \boxed{\underline{P} = 60^\circ \text{ (or) } \underline{QPR} = 60^\circ}$$

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