

CHAPTER 10

Circles

Theorem:

Statement:

If two tangents are drawn to a circle from an external point, then

- They subtend equal angles at the centre.
- They are equally inclined to the line segment, joining the centre to that point.

Given:

A circle with centre 'O', radius (r) and a point 'A' outside the circle such that AP and AQ are the tangents drawn to the circle from point 'A'.

To prove:

- $\angle AOP = \angle AOQ$
- $\angle OAP = \angle OAQ$

Proof:

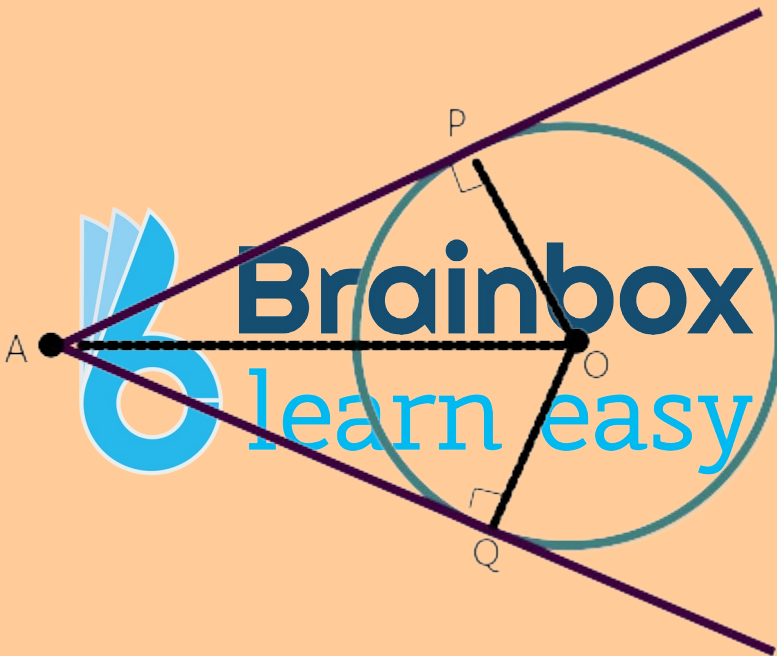
$$\angle OPA = \angle OQA = 90^\circ$$

($\because$ The tangent at any point of a circle is perpendicular to the radius through the point of contact)

In the right triangles OAP and OAQ,

$$AP = AQ$$

($\because$ Length of tangents drawn from an external point to a circle are equal)



$$OP = OQ \quad (\text{Radii of the same circle})$$

(and)

$$OA = OA \quad (\text{Common})$$

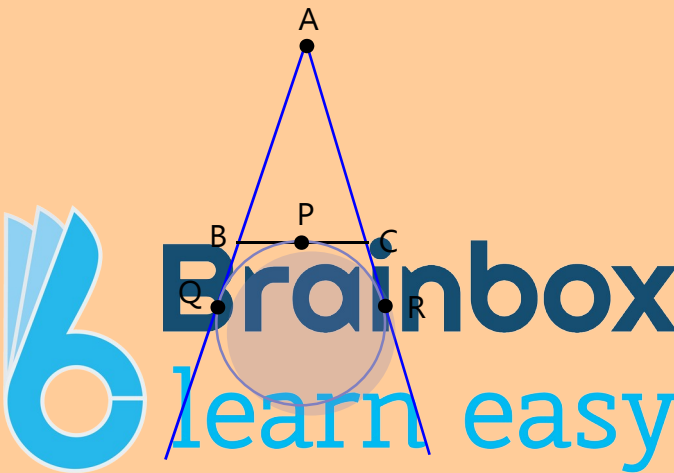
$$\therefore \triangle OAP \cong \triangle OAQ \quad (\text{By SSS congruency criterion})$$

$$\Rightarrow \underline{AOP} = \underline{AOQ} \text{ and } \underline{OAP} = \underline{OAQ} \quad (\text{By CPCT})$$

Example:

A circle is touching the side BC of a triangle ABC at P and touching AB and AC produced at Q and R respectively. Prove

that $AQ = \frac{1}{2}$ (Perimeter of $\triangle ABC$)



Sol.

$$2AQ = AQ + AR \quad (\because AQ = AR)$$

$$= (AB + BQ) + (AC + CR)$$

$$= AB + AC + (BQ + CR)$$

$$= AB + AC + (BP + CP) \quad (\because BQ = BP \text{ and } CR = CP)$$

$$2AQ = AB + AC + BC \quad (\text{Perimeter of } \triangle ABC)$$

$$\Rightarrow AQ = \frac{1}{2} (\text{Perimeter of } \triangle ABC)$$

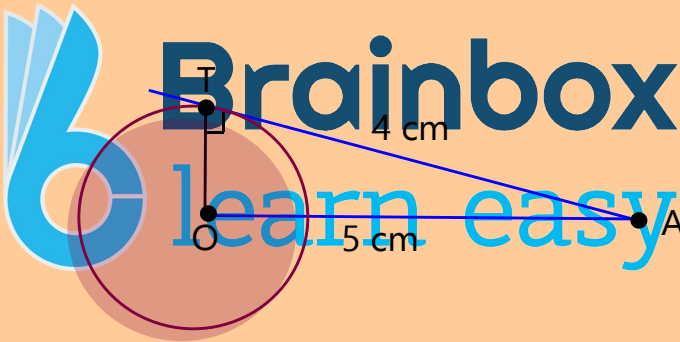
Example:

The length of a tangent from a point 'A' at distance 5 cm from the centre of the circle is 4 cm. Find the radius of the circle.

Sol.

The tangent to a circle is perpendicular to the radius through the point of contact.

$$\therefore \angle OTA = 90^\circ$$



Now, in the right $\triangle OTA$, we have

$$OA^2 = OT^2 + AT^2 \text{ (Pythagoras theorem)}$$

$$\Rightarrow 5^2 = OT^2 + 4^2$$

$$\Rightarrow OT^2 = 5^2 - 4^2$$

$$\Rightarrow OT^2 = (5 - 4)(5 + 4)$$

$$\Rightarrow OT^2 = (1)(9) = 9$$

$$\Rightarrow OT = \sqrt{9}$$

$$\therefore OT = 3$$

Hence, the radius of the circle is 3 cm.

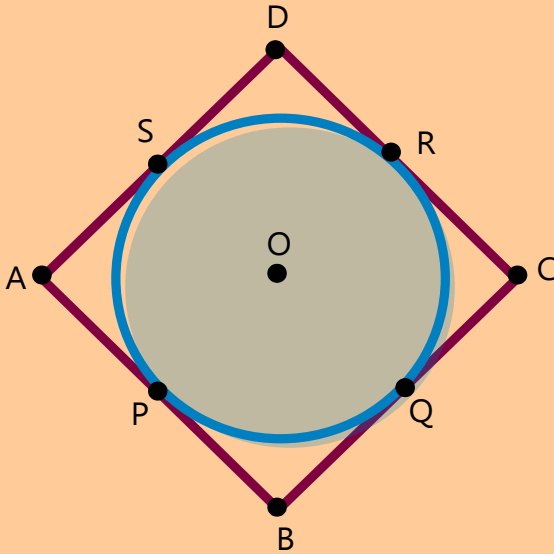
Example:

Prove that the parallelogram circumscribing a circle is a rhombus.

Sol.

We have ABCD, a parallelogram, which circumscribes a circle.
(i.e. Its sides touch the circle) with centre 'O'.

$\therefore$ Tangents to a circle from an external point are equal in length.



$$AP = AS$$

$$BP = BQ$$

$$CR = CQ$$

$$DR = DS$$

Adding, we get

$$(AP + BP) + (CR + DR) = (AS + DS) + (BQ + CQ)$$

$$\Rightarrow AB + CD = AD + BC$$

But $AB = CD$ (Opposite sides of parallelogram)

$$BC = AD$$

$$\therefore AB + CD = AD + BC$$

$$\Rightarrow 2AB = 2BC$$

$$\Rightarrow AB = BC$$

Similarly, $AB = DA$ and $DA = CD$

Thus, $AB = BC = CD = AD$

Hence, ABCD is a rhombus.

