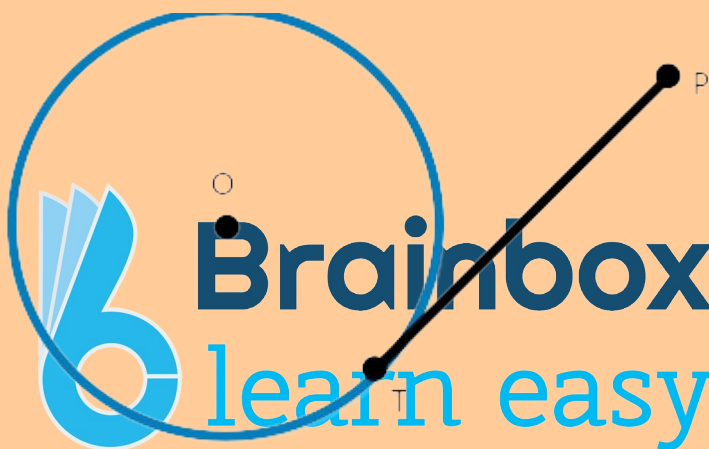


CHAPTER 10

Circles

Length of tangent:

If 'P' is an external point to the circle and PT is the tangent from 'P' to the circle, then $\overline{PT}$ is called the length of the tangent from 'P' to the circle.

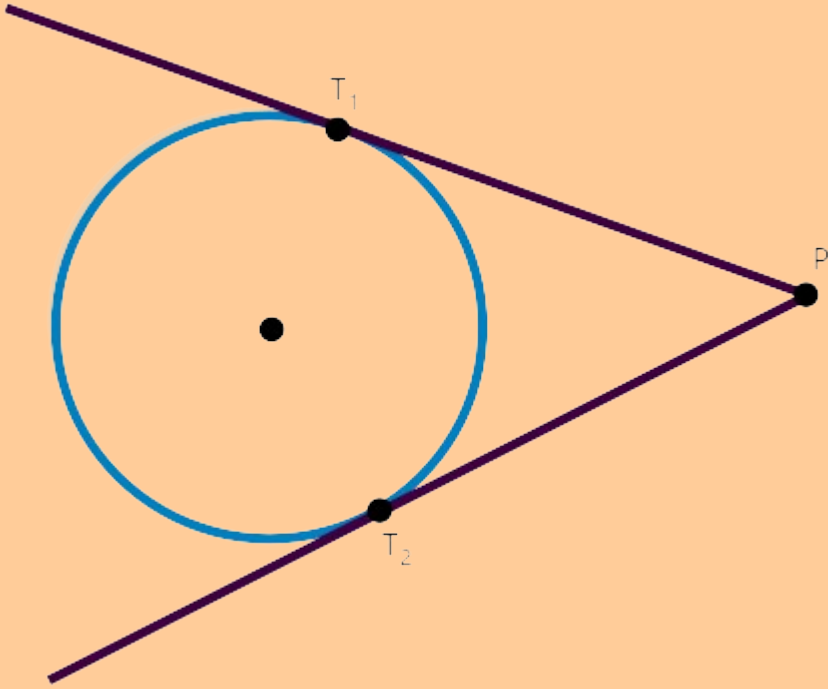


Example:

Here,

$\overline{PT_1}$ is the length of tangent

$\overline{PT_2}$ is the another length of tangent

**Theorem:****Statement:**

The lengths of tangents drawn from an external point to a circle are equal.

Given:

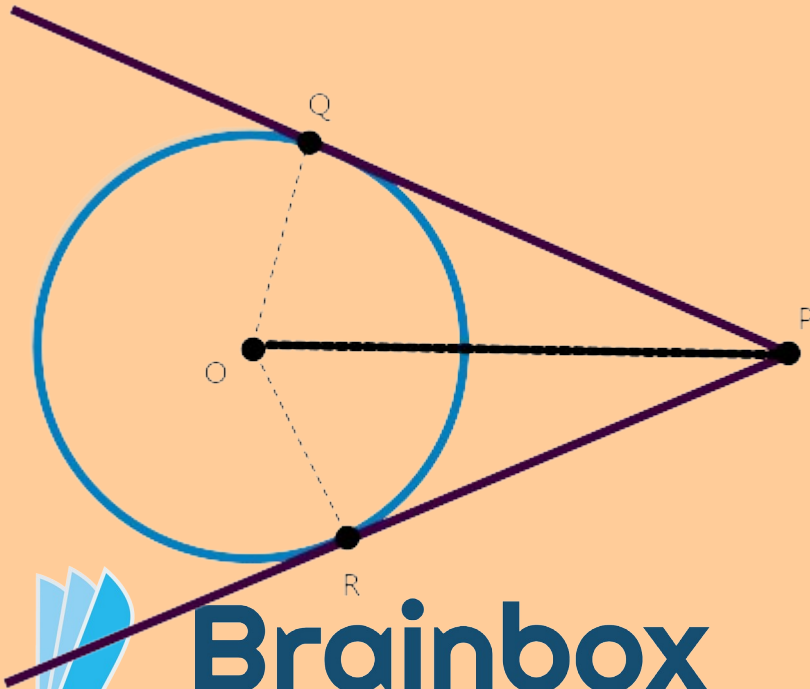
A circle with centre 'O' and a point 'P' lying outside the circle such that PQ and PR are the tangents to the circle from 'P'.

To prove:

$$PQ = PR$$

Construction:

Join OP, OQ and OR

**Proof:**

Brainbox
learn easy

$$\angle OQP \text{ and } \angle ORP = 90^\circ$$

($\because$ Tangent at any point of circle is perpendicular to the radius through the point of contact)

In $\triangle OQP$ and $\triangle ORP$,

$$OQ = OR \quad (\text{Radii of the same circle})$$

$$OP = OP \quad (\text{Common})$$

$$\angle OQP = \angle ORP \quad (\text{Each } 90^\circ)$$

$$\therefore \triangle OQP \cong \triangle ORP \quad (\text{By RHS congruency criterion})$$

$$\text{Hence, } PQ = PR \quad (\text{By C.P.C.T})$$

Note:

The line segment joining the external point to the centre of the circle bisects the angle between the two tangents.

Example:

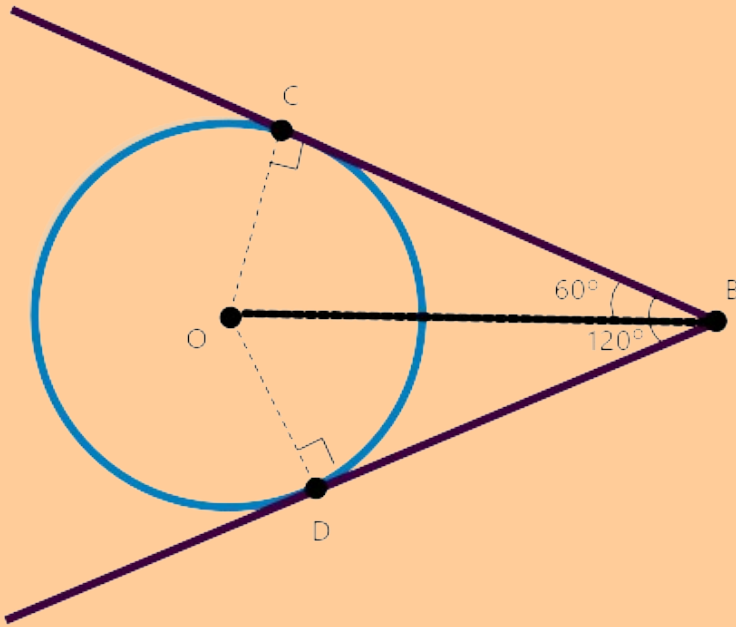
If from an external point B, two tangents BC and BD are drawn to a circle with centre 'O' such that $\angle DBC = 120^\circ$, prove that

$$BC + BD = BO \text{ i.e. } BO = 2BC$$

Sol.

Given,

$$\angle DBC = 120^\circ$$



Join OC, OD and BO

Since, BC and BD are tangents

$\therefore OC \perp BC$ and $OD \perp BD$

We know, OB is the angle bisector of $\angle DBC$

$\therefore \angle OBC = \angle OBD = 60^\circ$

In right angled $\triangle OBC$,

$$\cos 60^\circ = \frac{BC}{BO}$$

$$\Rightarrow \frac{1}{2} = \frac{BC}{BO}$$

$$\Rightarrow BO = 2 BC$$

Also, $BC = BD$

($\because$ Length of tangents drawn from an external point to circle are equal)

$$\therefore BO = BC + BC$$

$$\Rightarrow BO = BC + BD$$

Example:

In given figure, if $\angle PAO = 20^\circ$ then find $\angle POQ$.

Sol.

AP and AQ are tangents to the circle from an external point 'A'.

$$\text{So, } AP = AQ$$

In $\triangle APO$ and $\triangle AQO$,

$$AP = AQ \quad (\text{Proved})$$

$$OA = OA \quad (\text{Common})$$

$$OP = OQ \quad (\text{Radii})$$

$$\therefore \triangle APO \cong \triangle AQO \quad (\text{By SSS})$$

$$\text{So, } \angle PAO = \angle QAO = 20^\circ$$

$$\Rightarrow \angle PAQ = 40^\circ$$

and

$$\angle APO = \angle AQO = 90^\circ$$

Now, in quadrilateral APOQ

$$\angle PAQ + \angle AQD + \angle POQ + \angle APO = 360^\circ$$

$$\Rightarrow 40^\circ + 90^\circ + \angle POQ + 90^\circ = 360^\circ$$

$$\Rightarrow \angle POQ = 360^\circ - 220^\circ$$

$$\therefore \angle POQ = 140^\circ$$

