

CHAPTER 10

Circles

Theorem:

Statement:

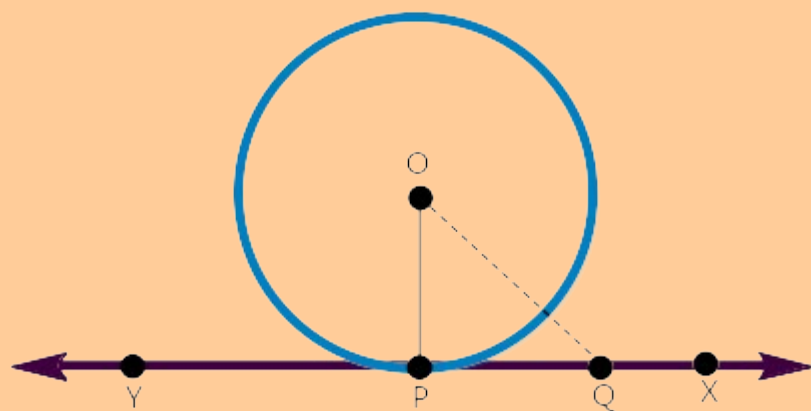
The tangent at any point of a circle is perpendicular to the radius through the point of contact.

Given:

A circle with centre 'O' and a tangent XY to the circle at a point 'P'.

To prove:

$OP \perp XY$



Proof:

- Take a point 'Q' on XY other than 'P' and join OQ.

- The point 'Q' must lie outside the circle.
- Therefore, OQ is longer than the radius OP of the circle.

i.e. $OQ > OP$

- This happens for every point on the line XY except the point 'P'.
- OP is the shortest of all the distances of the point 'O' to the points on XY. Hence, OP is perpendicular to XY.

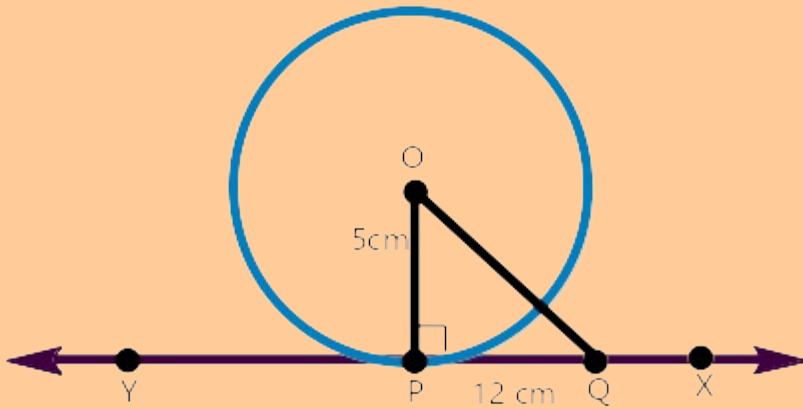
Example:

A tangent PQ at a point 'P' of a circle of radius 5cm meets a line through the centre 'O' at a point Q so that $PQ = 12$ cm. Find the length of OQ.

Sol.

Since, tangent at any point of a circle is perpendicular to the radius through the point of contact.

$\therefore OP \perp PQ \Rightarrow \triangle OPQ$ is a right angle triangle



In right angle OPQ,

$$OQ^2 = OP^2 + PQ^2$$

$$= 5^2 + 12^2$$

$$= 25 + 144$$

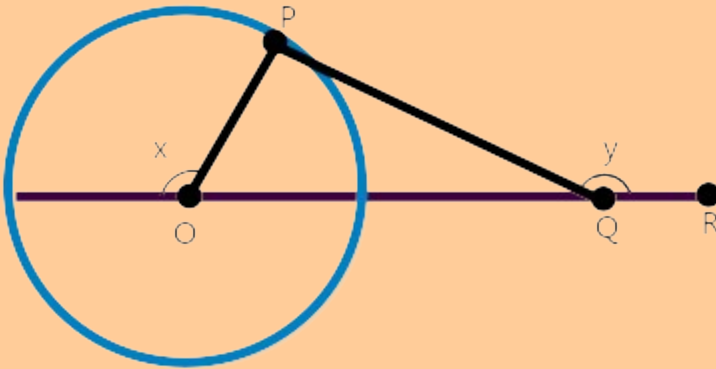
$$OQ^2 = 169$$

$$OQ = \sqrt{169} = 13 \text{ cm}$$

$$\therefore OQ = 13 \text{ cm}$$

Example:

From the given figure, find the value of $x + y$.



Sol.

$\therefore$ Tangent at any point of a circle is perpendicular to the radius through the point of contact.

$$\therefore OP \perp PQ \Rightarrow \angle OPQ = 90^\circ$$

In $\triangle OPQ$,

$$\angle PQR = \angle OPQ + \angle POQ \quad (\text{By exterior angle property})$$

$$\Rightarrow y = 90^\circ + (180^\circ - x)$$

$$\Rightarrow y = 270^\circ - x$$

$$\therefore x + y = 270^\circ$$