Chapter 02

POLYNOMIALS

Relationship between zeroes and co-efficients of a polynomial:

II. Let α, β, γ are the roots of cubic polynomial equation

$ax^3 + bx^2 + cx + d = 0$. Then

$$ax^3 + bx^2 + cx + d = a(x-\alpha)(x-\beta)(x-\gamma)$$

$$\Rightarrow ax^3 + bx^2 + cx + d$$

$$= a\left(x^3 - (\alpha+\beta)\gamma\right)x^2 + (\alpha\beta + \beta\gamma + \gamma\alpha)x - \alpha\beta\gamma$$

Comparing co-efficients, we have

$$\alpha + \beta + \gamma = -\frac{b}{a}$$

$$\alpha\beta + \beta\gamma + \alpha\gamma = \frac{c}{a}$$

$$\alpha\beta\gamma = -\frac{d}{a}$$

III. Let $\alpha, \beta, \gamma, \delta$ are roots of $ax^4 + bx^3 + cx^2 + dx + e = 0$,then

$\alpha + \beta + \gamma + \delta = -\frac{b}{a}$ (sum of roots taking single at a time).

$\alpha\beta + \alpha\gamma + \alpha\delta + \beta\gamma + \beta\delta + \gamma\delta = \frac{c}{a}$ (Sum of product taking two at a time).

$\alpha\beta\gamma + \beta\gamma\delta + \gamma\alpha\delta + \beta\gamma\delta = \frac{-d}{a}$ (Sum of product taking three at a time).

$\alpha\beta\gamma\delta = \frac{e}{a}$ (Product of roots).

NOTE:

In general, if $\alpha_1, \alpha_2, \alpha_3, ..., \alpha_n$ are the roots of polynomial: $a_0x^n + a_1x^{n-1} + a_2x^{n-2} + ... + a_{n-1}x + a_n = 0$

Then,

1. The sum of the roots is

$$\alpha_1 + \alpha_2 + \alpha_3 + ... + \alpha_n = S_1 = \sum \alpha = \frac{-a_1}{a_0}.$$

2. The sum of the product taken two at a time is

$$\alpha_1\alpha_2 + \alpha_1\alpha_3 + ... + \alpha_1 a_n + ...\alpha_{n-1}\alpha_n = S_2 = \sum \alpha_1\alpha_2 = \frac{a_2}{a_0}.$$

3. Sum of the product taken three at a time is

$$\alpha_1\alpha_2\alpha_3 + \alpha_1\alpha_2\alpha_4 + ... + \alpha_{n-2}.\alpha_{n-1}.\alpha_n = S_3 = \sum \alpha_1\alpha_2\alpha_3 = \frac{-a_3}{a_0}.$$

$\vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots$

n. Product of all roots = $S_n = (-1)^n \frac{a_n}{a_0}$.