

CHAPTER 05

Arithmetic Progressions

Middle term (s) of a finite A.P:

Let there be an A.P of 'n' terms. Also, let its first term is 'a' and common difference is 'd'. Now, two cases arises, which is shown in the following table.

When 'n' is ODD	When 'n' is EVEN
In this case, $\left(\frac{n+1}{2}\right)^{\text{th}}$ term is the middle term and is given by $a + \left(\frac{n+1}{2} - 1\right)d$	In this case, $\left(\frac{n}{2}\right)^{\text{th}}$ and $\left(\frac{n}{2} + 1\right)^{\text{th}}$ are the middle terms and are given by $a + \left(\frac{n}{2} - 1\right)d$ and $a + \left(\frac{n}{2} + 1 - 1\right)d = a + \frac{n}{2}d$ respectively.

Example:

Find the middle term(s) of the A.P. 7, 13, 19...205.

Sol.

Given A.P is 7, 13, 19...205.

Here, first term (a) = 7

Common difference (d) = $13 - 7 = 6$

Let there be 'n' terms in the given A.P.

$$\therefore a_n = 205$$

$$\Rightarrow a + (n - 1)d = 205$$

$$\Rightarrow 7 + (n - 1)6 = 205$$

$$\Rightarrow (n - 1)6 = 205 - 7 = 198$$

$$\Rightarrow n - 1 = \frac{198}{6} = 33$$

$$\Rightarrow n = 33 + 1 \Rightarrow n = 34$$

Here, 'n' is even so

$$\left(\frac{n}{2}\right)^{\text{th}} = \left(\frac{34}{2}\right)^{\text{th}} = 17^{\text{th}} \text{ and } \left(\frac{n}{2} + 1\right)^{\text{th}} = 17 + 1 = 18^{\text{th}} \text{ are middle terms}$$

and are given by,

$$a_{17} = a + (17 - 1)d = a + 16d = 7 + 16(6) = 103$$

and

$$a_{18} = a + (18 - 1)d = a + 17d = 7 + 17(6) = 109$$

Hence, middle term(s) are

$$a_{17} = 103 \text{ \& } a_{18} = 109$$