

CHAPTER 12**Areas Related to
Circles****Example:**

Two circles touch each other externally so that the distance between their centres is equal to the sum of their radii. The sum of their areas is $208\pi\text{cm}^2$ and the distance between their centres is 20cm. Find the radii of the circles.

Sol.

Let the radii of two circles be $r_1\text{cm}$ and $r_2\text{cm}$.

Given that,

Distance between the centres of the circles = 20cm

$$\Rightarrow r_1 + r_2 = 20\text{ cm} \dots\dots\dots (1)$$

Sum of the areas of the two circles = $208\pi\text{cm}^2$

$$\Rightarrow \pi r_1^2 + \pi r_2^2 = 208\pi$$

$$\Rightarrow \cancel{\pi} (r_1^2 + r_2^2) = 208\cancel{\pi}$$

$$\Rightarrow (r_1^2 + r_2^2) = 208 \dots\dots\dots (2)$$

Now,

$$(r_1 + r_2)^2 + (r_1 - r_2)^2 = 2(r_1^2 + r_2^2)$$

From (1) & (2),

$$(20)^2 + (r_1 - r_2)^2 = 2(208)$$

$$400 + (r_1 - r_2)^2 = 416$$

$$(r_1 - r_2)^2 = 416$$

$$(r_1 - r_2)^2 = 416 - 400$$

$$(r_1 - r_2)^2 = 16$$

$$r_1 - r_2 = \pm\sqrt{16}$$

$$r_1 - r_2 = \pm 4$$

$$r_1 - r_2 = 4 \quad (\text{or}) \quad r_1 - r_2 = -4 \dots\dots (3)$$

By adding (1) & (3), we get

$$2r_1 = 20 + 4$$

$$2r_1 = 24$$

$$r_1 = \frac{24}{2}$$

$$r_1 = 12$$

(OR)

$$2r_1 = 20 - 4$$

$$2r_1 = 16$$

$$r_1 = \frac{16}{2}$$

$$r_1 = 8$$

$$\Rightarrow r_1 = 12 \text{ (or) } 8 \text{ cm}$$

From (1),

$$12 + r_2 = 20$$

$$r_2 = 20 - 12$$

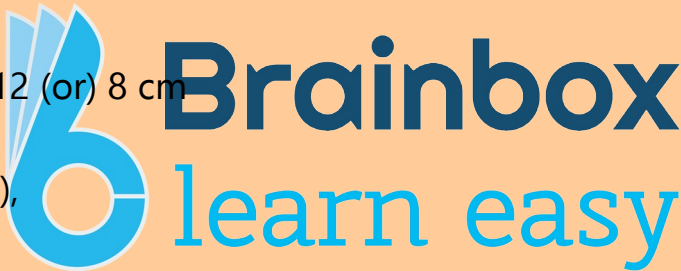
$$r_2 = 8 \text{ cm}$$

(OR)

$$8 + r_2 = 20$$

$$r_2 = 20 - 8$$

$$r_2 = 12$$



Hence, the radii of two circles are 8 cm and 12 cm.

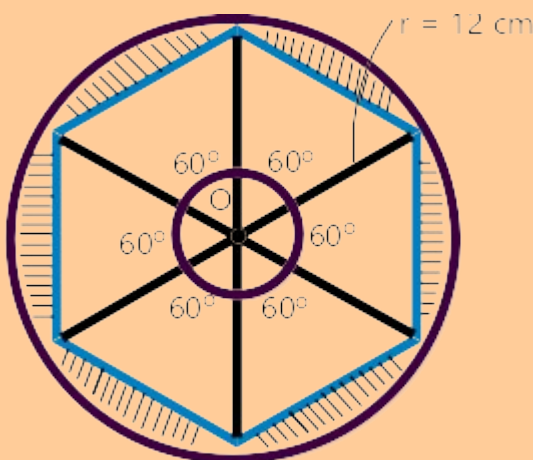
Example:

Around table has 6 equal designs as shown in the figure. If the radius of the top is 14 cm. Find the cost of making the designs

with paint at the rate of 7.5 per cm^2 (Use $\sqrt{3} = 1.732$).

Sol.

In the question, around table cover is given, which has six equal designs.



Let us join ends of the design, since the designs are equal.

We know that, the radius of circumscribing circle of a regular hexagon is equal to the length of its side.

∴ Each side of regular hexagon = 14 cm

Therefore area of six design segments =

$$(\text{Area of circle}) - (\text{Area of the regular hexagon}) =$$

Now, Area of circle = πr^2

$$= \frac{22}{7} \times 14^2 \times 14$$

$$= 616 \text{ cm}^2 \dots\dots\dots (1)$$

Area of regular hexagon = $6 \times \frac{\sqrt{3}}{4} a^2$

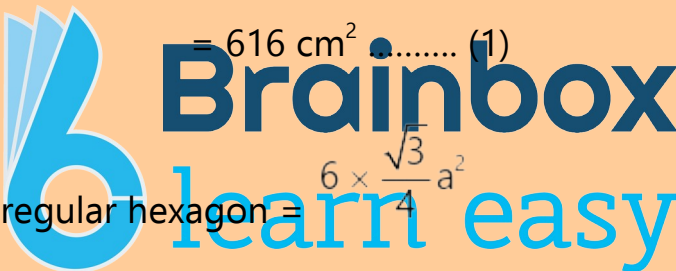
$$= 6 \times \frac{\sqrt{3}}{4} \times 14^2 \times 14$$

$$= 6 \times \sqrt{3} \times 7 \times 7$$

$$= 509.2 \text{ cm}^2 \dots\dots\dots (2)$$

Hence, area of six designs = 616 – 509.21 (From (1) & (2))

$$= 106.79 \text{ cm}^2$$



∴ Cost of painting the design at the rate of 5 Rs. per cm^2 is equal to 106.79×5 .

$$= 533.95 \text{ Rs.}$$

∴ Cost of painting the design = 533.95 Rs.

