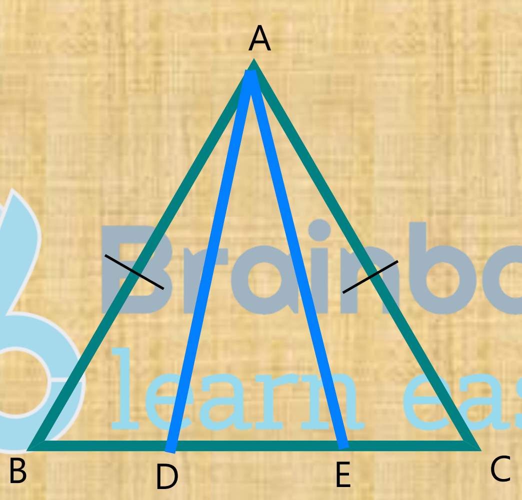
CHAPTER 07

Triangles

Examples:

Example 1:

In an isosceles triangle ABC with AB = AC; D and E are points on BC such that BE = CD. Show that AD = AE.



Sol.

BE = CD

BE – DE = CD – DE

$\Rightarrow$ BD = CE

In ΔABD and ΔACE, we have

AB = AC (Given)

$\angle B = \angle C$ (Angles opposite to equal sides)

$BD = CE$ (Proved above)

$\Delta ABD \cong \Delta ACE \quad (\because$ SAS congruence)

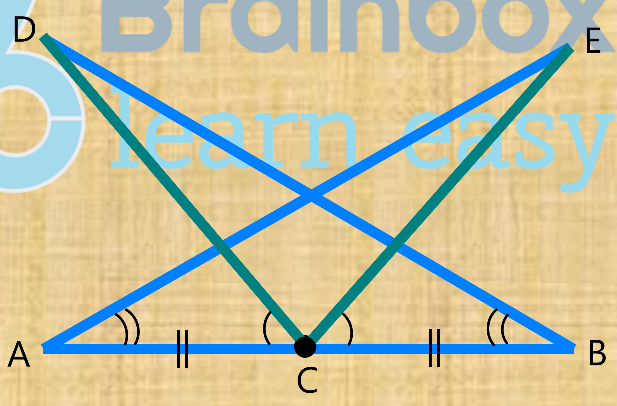
$\Rightarrow AD = AE$ ($\because$ C.P.C.T)

Example 2:

Use the information given in the adjoining figure to prove

(i) $\Delta BDC \cong \Delta EAC$

(ii) $DC = EC$



Sol.

I) Let $\angle ACD = \angle BCE = x^{\circ}$

$\angle ACE = \angle DCE + \angle ACD = \angle DCE + x$ .... (1)

$\angle BCD = \angle DCE + \angle BCE = \angle DCE + x$ .... (2)

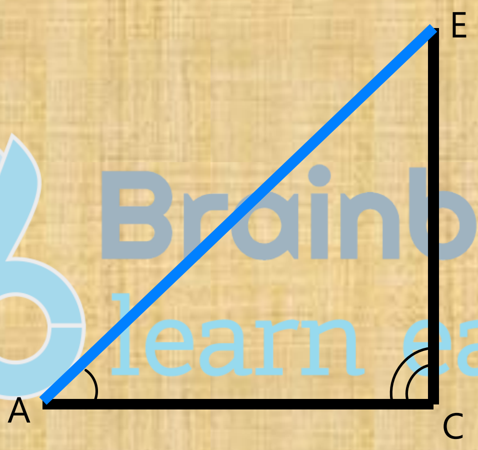
From (1) & (2),

We get,

$\angle ACE = \angle BCD$

II) In ΔDBC and ΔEAC

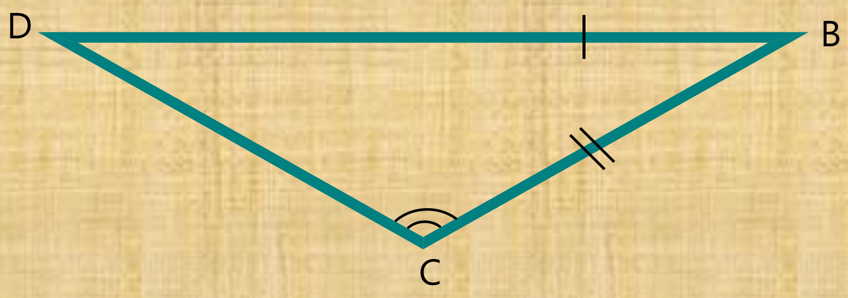
$\angle ACE = \angle BCD$ (Proved above)



$BC = AC$ (Given)

$\angle CBD = \angle CAE$ (Given)

$\Delta DBC \cong \Delta EAC$ (By S.A.S)



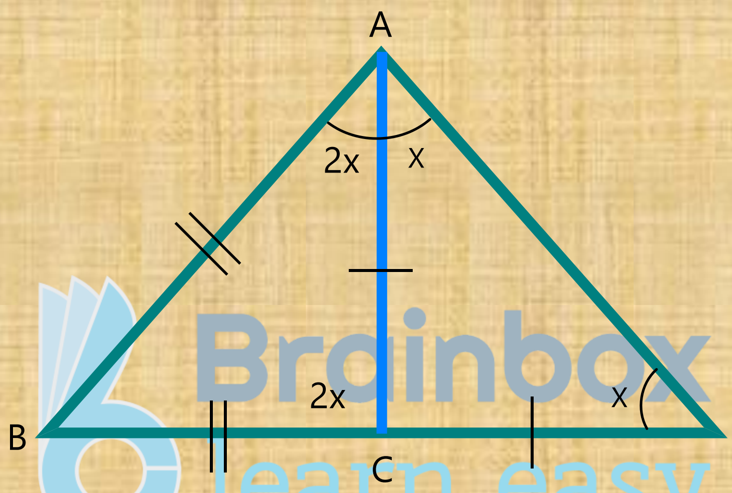
$\therefore\ \Delta DBC \cong \Delta EAC$

DC = EC (C.P.C.T)

Example 3:

In the adjacent figure, AB = BC and AC = CD. Prove that

$\angle BAD : \angle ADB = 3 : 1.$



Sol.

Let $\angle ADB = x^\circ$

In ΔACD, AC = CD

$\Rightarrow \angle CAD = \angle CDA = x$ and

Exterior angle = Sum of 2 interior angles

$\angle ACD = \angle CAD + \angle CDA = x^\circ + x^\circ = 2x^\circ$

$\angle BAC = \angle ACB = 2x^\circ$ (In ΔABC, AB = BC)

$\angle BAD = \angle BAC + \angle CAD$

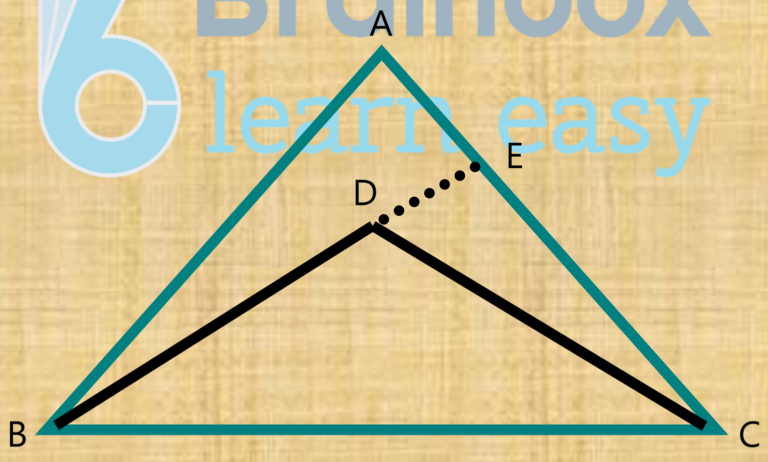
$\angle BAD = 2x + x = 3x$

$$\frac{\angle BAD}{\angle ADB} = \frac{3x}{x} = \frac{3}{1}$$

$\therefore \angle BAD : \angle ADB = 3:1$

Example 4:

In the given figure, ABC is a triangle and D is any point in its interior. Show that BD + DC < AB + AC.



Sol.

Produce BD.

$\therefore$ Sum of two sides is greater than the third side.

In ΔABE, AB + AE > BE

AB + AE > BD + DE .... (1)

In ΔCDE,

DE + EC > DC .... (2)

Adding (1) and (2), we get

AB + AE+ DE + EC > BD + DE + DC

AB + (AE + EC) > BD + DC

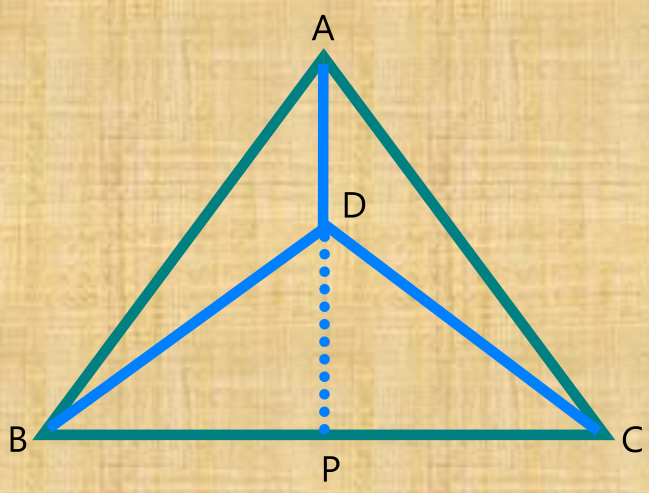
$\Rightarrow$ BD + DC < AB + AC

Example 5:

In ΔABC, ΔDBC are two isosceles triangles on the same base BC and vertices A & D are on the same side of BC. If AD is extended to intersect BC at P. Show that,

I) $\Delta ABD \cong \Delta ACD$

II) $\Delta ABP \cong \Delta ACP$



Sol.

I) In ΔABP and ΔACP, we have

AB = AC (Given)

AD = AD (Given)

BD = CD (Given)

$\Delta ABD \cong \Delta ACD$ (By SSS congruence rule)

$\Rightarrow \angle BAD = \angle CAD$ (By C.P.C.T)

II) In ΔABP and ΔACP, we have

AB = AC (Given)

$\angle BAP = \angle CAP$ (Proved in 1^{st})

AP = AP

$\therefore\ \Delta ABD \cong \Delta ACD$ (SAS congruence rule)