CHAPTER 08

Quadrilaterals

Properties of parallelogram:

- A diagonal of a parallelogram divides it into two congruent triangles.
- A quadrilateral is a parallelogram if it's opposite sides are equal.
- The opposite angles of a parallelogram are equal.
- A quadrilateral is a parallelogram if it's opposite angles are equal.
- The diagonals of a parallelogram bisect each other.
- If the diagonals of a quadrilateral bisect each other than, the quadrilateral is a parallelogram.
- A quadrilateral is a parallelogram if one pair of opposite sides are equal and parallel.

Midpoint of sides of a triangle/Midpoint theorem:

Statement of midpoint theorem:

The line segment joining the midpoints of any two sides of a triangle is parallel to the third side and equal to half of it.

Given that, in a triangle ABC, D is the midpoint of side AB and E is the midpoint of side AC. Join DC.

To prove:

- DE || BC and DE = $\frac{1}{2}$BC

Construction:

Produce line segment DE to F such that DE = EF. Join FC.

Proof:

In ΔAED and ΔCEF, we have

AE = EC [E is midpoint of AC]

$\angle AED = \angle CEF$ [V.O.A]

DE = EF [By construction]

$\therefore\ \Delta AED \cong \Delta CEF$ [By SAS congruency]

Hence, AD = CF --------------- (1)

$\angle ADE = \angle CFE$ (By C.P.C.T) ---------- (2)

DB = FC (As AD = CF and AD = BD) ---------- (3)

$\angle ADE = \angle CFE$ (DF is a transversal line)

$\therefore$ AD || FC $\Rightarrow$ DB || FC ----------- (4)

From (3) & (4)

BCFD is a parallelogram

Hence, DF || BC and DF = BC (Opp. sides of parallelogram)

But,

DE = EF (By construction)

$$\Rightarrow DE = \frac{1}{2}DE \qquad \text{From equation (1)}$$

$$DE = \frac{1}{2}BE$$

Hence, DE || BC and DE = $\frac{1}{2}$BC.

Can you state the converse of the above theorem? Is the converse true?

The line drawn through the midpoint of one side of a triangle parallel to another side intersects the third side at its midpoint.

Given that,

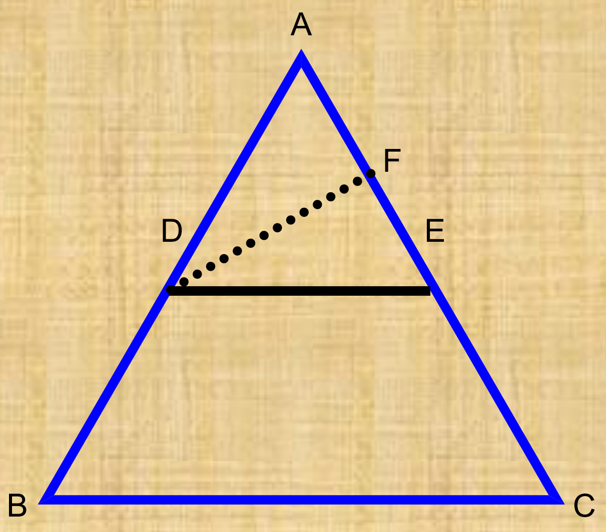
ΔABC, In which D is the midpoint of AB and DE || BC.

To prove that E is the midpoint of AC.

Proof:

Let if possible E is not midpoint of AC.

Draw a line DF from a point D intersecting AC at F such that F is the midpoint of AC.



In ΔABC, D is the midpoint of AB (Given) and F is the midpoint of AC.

$\therefore$ By midpoint theorem,

DF || BC ……………. (1)

DE || BC ……………. (2)

From (1) & (2),

Two intersecting lines DE and DF are both parallel to the line BC.

This contradicts the parallel line axiom.

So our assumption is wrong.

Hence, E is the midpoint of AC.