CHAPTER 01

Number System

Rationalising real numbers:

We can simplify the given expression whose denominator is irrational (or contains square root). To convert the denominators in such cases free from square roots, we multiply both the numerator and denominator of the given number by a suitable factor. This factor is called the rationalising factor and this process is known as rationalising the denominator.

Some general rationalising factors:

If a & b are integers, x & y are natural numbers then,

S. No	Number	Rationalising Factor(R.F)
1	$a+\sqrt{b}$	$a-\sqrt{b}$
2	$a+b\sqrt{x}$	$a-b\sqrt{x}$
3	$\sqrt{a}+\sqrt{b}$	$\sqrt{a}-\sqrt{b}$
4	$\frac{a}{\sqrt{x}}$	$\sqrt{x}$
5	$\frac{a}{\left(a\pm b\sqrt{x}\right)}$	$\left(a\mp b\sqrt{x}\right)$
6	$\frac{1}{\left(a\sqrt{x}\pm b\sqrt{y}\right)}$	$a\sqrt{x}\mp b\sqrt{y}$
7	$\sqrt[3]{a}\pm\sqrt[3]{b}$	$a^{2/3}\mp a^{1/3}b^{1/3}+b^{2/3}$

8	$\sqrt{a}+\sqrt[3]{b}$	$\left(\sqrt{a}-\sqrt[3]{b}\right)\left(a^2+a\sqrt[3]{b^2}+b\sqrt[3]{b}\right)$
	Note: Conjugate of a surd is R.F but the R.F need not be a conjugate.	

Example:

Rationalise the denominator of $\frac{1}{7+3\sqrt{2}}$.

Sol.

Given, $\frac{1}{7+3\sqrt{2}}$

On rationalising, we get

$$=\frac{1}{7+3\sqrt{2}}\times\left(\frac{7-3\sqrt{2}}{7-3\sqrt{2}}\right)$$

$$=\frac{7-3\sqrt{2}}{(7)^2-\left(3\sqrt{2}\right)^2}\ \left(\because(a+b)(a-b)=a^2-b^2\right)$$

$$=\frac{7-3\sqrt{2}}{49-18}$$

Rationalise of $\frac{1}{7+3\sqrt{2}}=\frac{7-3\sqrt{2}}{31}$

Example:

If $x = 5 + 2\sqrt{6}$, then find the value of $x + \frac{1}{x}$.

Sol.

Given,

$$x = 5 + 2\sqrt{6}$$

Now,

$$\frac{1}{x} = \frac{1}{5+2\sqrt{6}} \times \frac{5-2\sqrt{6}}{5-2\sqrt{6}} = \frac{5-2\sqrt{6}}{(5)^2 - \left(2\sqrt{6}\right)^2} = \frac{5-2\sqrt{6}}{25-24}$$

$$\Rightarrow \frac{5-2\sqrt{6}}{1} = 5 - 2\sqrt{6}$$

$$\therefore \; x + \frac{1}{x} = 5 + 2\sqrt{6} + 5 - 2\sqrt{6} = 10$$

Example:

If 'a' and 'b' are two rational numbers and

$\frac{1}{7+4\sqrt{3}} + \frac{1}{2+\sqrt{5}} = a - 4\sqrt{b} + \sqrt{5}$, then find the values of 'a' and 'b'.

Sol.

The rationalising factor of $7 + 4\sqrt{3}$ is $7 - 4\sqrt{3}$ and

The rationalising factor of $2 + \sqrt{5}$ is $2 - \sqrt{5}$

$$\frac{1}{7+4\sqrt{3}} + \frac{1}{2+\sqrt{5}} = \left(\frac{1}{7+4\sqrt{3}} + \frac{7-4\sqrt{3}}{7-4\sqrt{3}}\right) + \left(\frac{1}{2+\sqrt{5}} \times \frac{2-\sqrt{5}}{2-\sqrt{5}}\right)$$

$$= \left(\frac{7-4\sqrt{3}}{(7)^2-\left(4\sqrt{3}\right)^2} \right) + \left(\frac{2-\sqrt{5}}{(2)^2-\left(\sqrt{5}\right)^2} \right)$$

$$= \frac{7-4\sqrt{3}}{49-48} + \frac{2-\sqrt{5}}{4-5}$$

$$= \frac{7-4\sqrt{3}}{1} + \frac{2-\sqrt{5}}{(-1)}$$

$$= 7-4\sqrt{3}-2+\sqrt{5}$$

$$= 5-4\sqrt{3}+\sqrt{5}$$

$$= a-4\sqrt{b}+\sqrt{5}$$

On comparing, we get $\boxed{a=5}$ and $\boxed{b=3}$