# CHAPTER 01

# Number System

**Conversion of non – terminating repeating decimal number into the form (p/q):**

**Step I:**

Put the given decimal number equal to ‘x’.

**Step II:**

Remove the bar if any and write the repeating digits at least twice.

**Step III:**

If the repeating decimal has 1 place repetition, multiply it by 10, if there is 2 place repetition multiply it by 100 and so on.

**Step IV:**

Subtract the number in step (II) from the number obtained in step (III).

**Step V:**

Divide both sides of the equation by the co-efficient of ‘x’.

**Example:**

Express the following recurring decimal expansions in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$.

(I) $0.\overline{5}$ (II) $0.2\overline{37}$

**Sol.**

**(I)** Let $x = 0.\bar{5}$,

$$\Rightarrow x = 0.5555.......(1)$$

Multiplying (1) by 10 both sides, we get

$$10 \times x = 10 \times \{0.5555......\}$$

$$\Rightarrow 10x = 5.555......(2)$$

Subtracting (1) from (2), we get

$$\begin{array}{rcl} 10x & = & 5.555....... \\ x & = & 0.555....... \\ (-) & & (-) \\ \hline 9x & = & 5 \end{array}$$

$$\Rightarrow x = \frac{5}{9}$$

Thus, $0.\bar{5} = \frac{5}{9}$

**(II)** Let $x = 0.2\overline{37}$

$$\Rightarrow x = 0.2373737......(1)$$

Multiplying (1) by 100 both sides, we get

$$\Rightarrow 100 \times x = 100 \times 0.2373737$$

$$\Rightarrow 100\ x = 23.73737......(2)$$

Subtracting (1) from (2), we get

100x = 23.737373.......

x = 0.237373.......

(-) (-)

99x = 23.5

$$\Rightarrow x = \frac{23.5}{99} \times \frac{10}{10}$$

$$\Rightarrow x = \frac{235}{990}$$

$$\therefore x = \frac{47}{198}$$

Thus, $0.2\overline{37} = \frac{47}{198}$

**Decimal expansion of irrational numbers:**

The decimal expansion of irrational numbers is non – terminating non – repeating.

**Methods of finding irrational numbers between two rational numbers:**

**Method I:**

Let $\frac{a}{b}$ and $\frac{c}{d}$ be any two rational numbers. To find an irrational number between $\frac{a}{b}$ and $\frac{c}{d}$, firstly write $\frac{a}{b}$ and $\frac{c}{d}$ in their decimal form.

**Step 1:**

Now, as we know, irrational number is non – terminating non – repeating decimal we can find the required number of irrational numbers.

**Method II:**

If ‘a’ and ‘b’ are two positive rational numbers such that ‘ab’ is not a perfect square of a rational number, then $\sqrt{ab}$ is an irrational number lying between ‘a’ and ‘b’.

**Example:**

Find any two irrational numbers between $\frac{1}{5}$ and $\frac{2}{7}$.

**Sol.**



To find the irrational numbers, firstly we will divide 1 by 5 and 2 by 7.

Now,

$$\begin{array}{rcl} 5\,\Big) & 1.0 & \Big(\,0.2 \\ & (-)\,1.0 & \\ \hline & 0 & \\ \hline \end{array}$$

$$\therefore \frac{1}{5} = 0.2$$

$$\frac{2}{7} = 0.\overline{285714}$$

$$
\begin{array}{r|l}
7 & 2.000000 \quad 0.285714\ldots \\
 & (-)\ 14 \\ \hline
 & \quad 60 \\
 & (-)\ 56 \\ \hline
 & \quad 40 \\
 & (-)\ 35 \\ \hline
 & \quad 50 \\
 & (-)\ 49 \\ \hline
 & \quad 10 \\
 & (-)\ \ 7 \\ \hline
 & \quad 30 \\
 & (-)\ 28 \\ \hline
 & \quad\ \ 2
\end{array}
$$

Thus, the required irrational number will lie between 0.2 and $0.\overline{285714}$. Also, the irrational numbers have non – terminating non – repeating decimals. Hence, two such irrational numbers are

**Examples:**

0.201201120111….

0.24114111411114…

0.25025002500025….

**Example:**

Find an irrational number between 2 and 4.

**Sol.**

An irrational number between 2 and 4 is $\sqrt{2 \times 4}$ (From method II)

$$= \sqrt{2} \times 2 = 2\sqrt{2}$$