

CHAPTER 01

Number System

Conversion of non – terminating repeating decimal number into the form (p/q) :

Step I:

Put the given decimal number equal to 'x'.

Step II:

Remove the bar if any and write the repeating digits at least twice.

Step III:

If the repeating decimal has 1 place repetition, multiply it by 10, if there is 2 place repetition multiply it by 100 and so on.

Step IV:

Subtract the number in step (II) from the number obtained in step (III).

Step V:

Divide both sides of the equation by the co-efficient of 'x'.

Example:

Express the following recurring decimal expansions in the form $\frac{p}{q}$,

where p and q are integers and $q \neq 0$.

(I) $0.\bar{5}$ (II) $0.2\bar{37}$

Sol.

(I) Let $x = 0.\bar{5}$,

$$\Rightarrow x = 0.5555.....(1)$$

Multiplying (1) by 10 both sides, we get

$$10 \times x = 10 \times \{0.5555.....\}$$

$$\Rightarrow 10x = 5.555.....(2)$$

Subtracting (1) from (2), we get

$$10x = 5.555.....$$

$$x = 0.555.....$$

$$\begin{array}{r} (-) \quad (-) \\ 10x = 5.555..... \\ - \quad x = 0.555..... \\ \hline 9x = 5 \end{array}$$

$$\Rightarrow x = \frac{5}{9}$$

$$\text{Thus, } 0.\bar{5} = \frac{5}{9}$$

(II) Let $x = 0.2\bar{37}$

$$\Rightarrow x = 0.2373737.....(1)$$

Multiplying (1) by 100 both sides, we get

$$\Rightarrow 100 \times x = 100 \times 0.2373737$$

$$\Rightarrow 100x = 23.73737.....(2)$$

Subtracting (1) from (2), we get

$$100x = 23.737373.....$$

$$x = 0.237373.....$$

$$\begin{array}{r} (-) \quad (-) \\ \hline \end{array}$$

$$99x = 23.5$$

$$\Rightarrow x = \frac{23.5}{99} \times \frac{10}{10}$$

$$\Rightarrow x = \frac{235}{990}$$

$$\therefore x = \frac{47}{198}$$

$$\text{Thus, } 0.\overline{237} = \frac{47}{198}$$

Decimal expansion of irrational numbers:

The decimal expansion of irrational numbers is non – terminating non – repeating.

Methods of finding irrational numbers between two rational numbers:

Method I:

Let $\frac{a}{b}$ and $\frac{c}{d}$ be any two rational numbers. To find an irrational

number between $\frac{a}{b}$ and $\frac{c}{d}$, firstly write $\frac{a}{b}$ and $\frac{c}{d}$ in their decimal form.

Step 1:

Now, as we know, irrational number is non – terminating non – repeating decimal we can find the required number of irrational numbers.

Method II:

If ‘a’ and ‘b’ are two positive rational numbers such that ‘ab’ is not a perfect square of a rational number, then $\sqrt{ab}$ is an irrational number lying between ‘a’ and ‘b’.

Example:

Find any two irrational numbers between $\frac{1}{5}$ and $\frac{2}{7}$.

Sol.

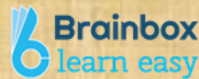
To find the irrational numbers, firstly we will divide 1 by 5 and 2 by 7.

Now,

$$\begin{array}{r} 5 \overline{) 1.0} \quad (0.2 \\ \underline{(-) 1.0} \\ 0 \end{array}$$

$$\therefore \frac{1}{5} = 0.2$$

$$\frac{2}{7} = 0.\overline{285714}$$



$$\begin{array}{r}
 7 \overline{) 2.000000} \quad \left(0.285714... \right. \\
 \underline{(-) 14} \\
 60 \\
 \underline{(-) 56} \\
 40 \\
 \underline{(-) 35} \\
 50 \\
 \underline{(-) 49} \\
 10 \\
 \underline{(-) 7} \\
 30 \\
 \underline{(-) 28} \\
 2
 \end{array}$$

Thus, the required irrational number will lie between 0.2 and 0.285714. Also, the irrational numbers have non – terminating non – repeating decimals. Hence, two such irrational numbers are

Examples:

0.201201120111....

0.24114111411114...

0.25025002500025....

Example:

Find an irrational number between 2 and 4.

Sol.

An irrational number between 2 and 4 is $\sqrt{2 \times 4}$ (From method II)

$$= \sqrt{2} \times 2 = 2\sqrt{2}$$

