CHAPTER 01

Number System

Irrational numbers:

A number that cannot be expressed in the form $\frac{p}{q}$ (where $q \neq 0$), p and q are integers is an irrational number or any non – terminating and non – recurring decimal is an irrational number.

Example:

$\sqrt{2}, \sqrt{3}, \sqrt{5}$, $0.101001000100001\ldots, \pi$ etc. cannot be expressed in the form $\frac{p}{q}$ $(q \neq 0)$, where p and q are integers.

$\therefore$ These numbers are irrational numbers.

Note:

- There are infinitely many irrational numbers between any two irrational numbers.
- We often take = $\frac{22}{7}$ as an approximate value of π, but $\pi \neq \frac{22}{7}$. Actually $\pi = 3.14159265358979323384626433832795\ldots$ So, π is an irrational number.

Representation of irrational numbers on number line:

To represent $\sqrt{n}$ on the number line, where $n \in N$ we have to follow the following steps.

Step 1:

Write 'n' as the sum of squares of two numbers.

i.e.. $n = a^2 + b^2$

Example:

- If n = 3, then $3 = 1^2 + \left(\sqrt{2}\right)^2$
- If n = 5, then $5 = 1^2 + 2^2$

Step 2:

Draw OA = a units on number line.

Step 3:

Draw AB = b units, such that $AB \perp OA$. Join OB.

Step 4:

Taking 'O' as centre and OB as radius draw an arc, which cuts the number line at C, where point 'C' represents the required irrational number on number line.

Note:

- In the same way, we can locate $\sqrt{n}$ for any positive integer n, after locating $\sqrt{n-1}$ on the number line.
- If 'n' is a natural number other than a perfect square then $\sqrt{n}$ is an irrational number.

Example:

Represent each of the numbers $\sqrt{2}$ and $\sqrt{3}$ on the number line.

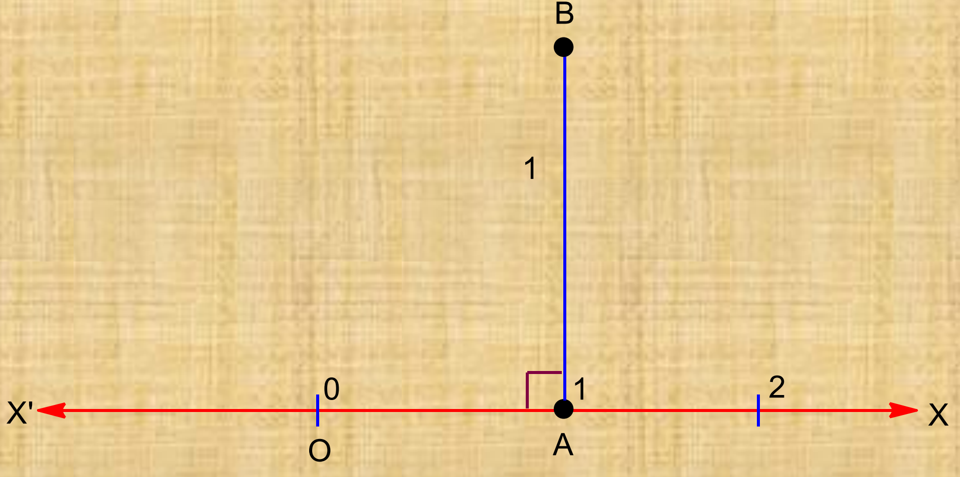
Sol.

Step 1:

Draw a number line $X'OX$ and let 'O' be the origin. Take 'A' on it. Points 'O', A 'represents numbers 'O', and '1' respectively.

Step 2:

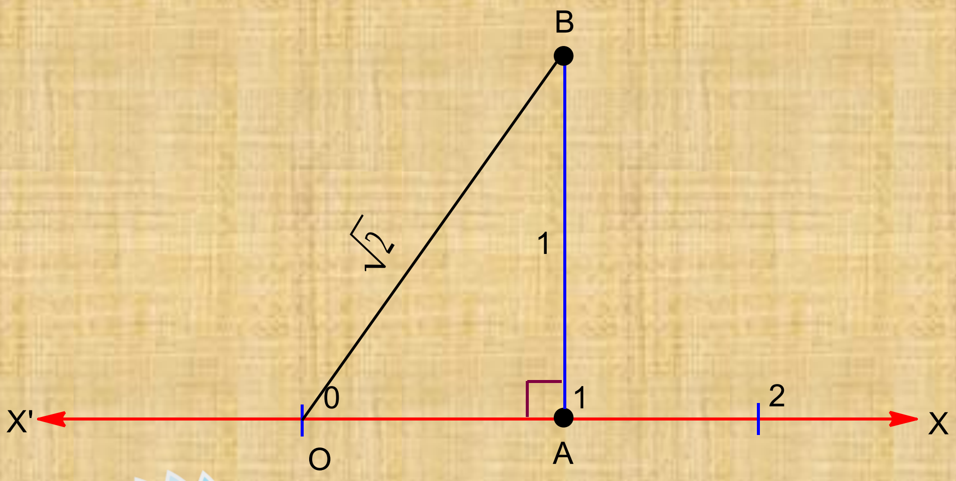
Take OA = 1 unit and draw AB = 1 unit such that $AB \perp OA$.



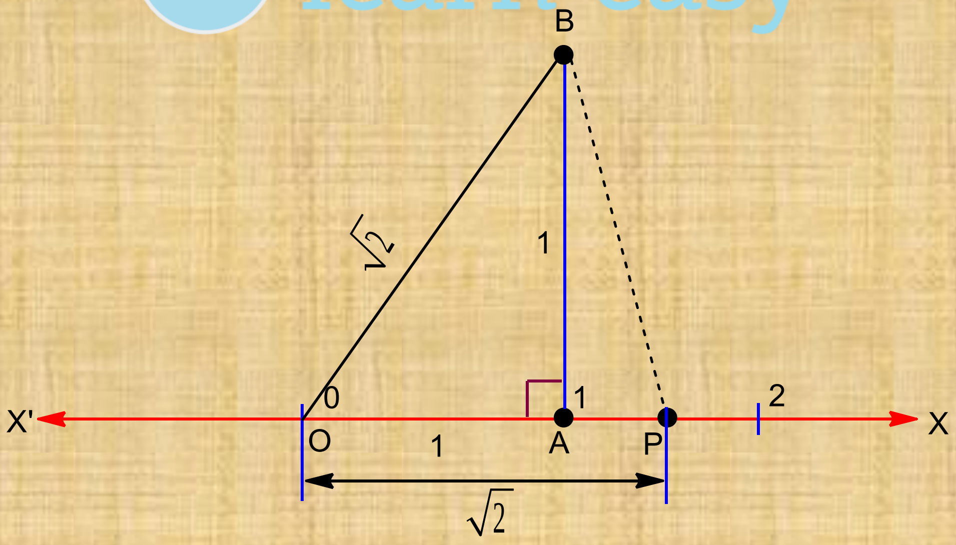
Step 3:

Join O, B. We get $OB = \sqrt{OA^2 + AB^2} = \sqrt{1^2 + 1^2} = \sqrt{2}$ units.

Step 4:



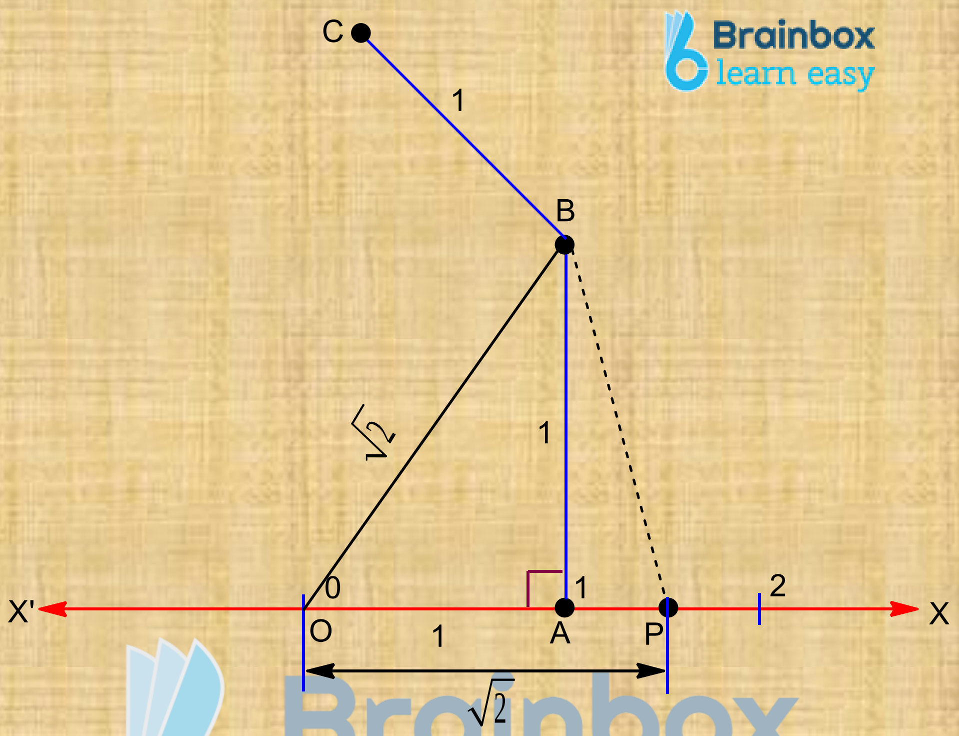
With ‘O’ as centre and radius, $OB = \sqrt{2}$ units draw an arc, which intersects OX at P. Then OP = OB = $\sqrt{2}$ units.



Thus, the point ‘P’ represents $\sqrt{2}$ on the number line.

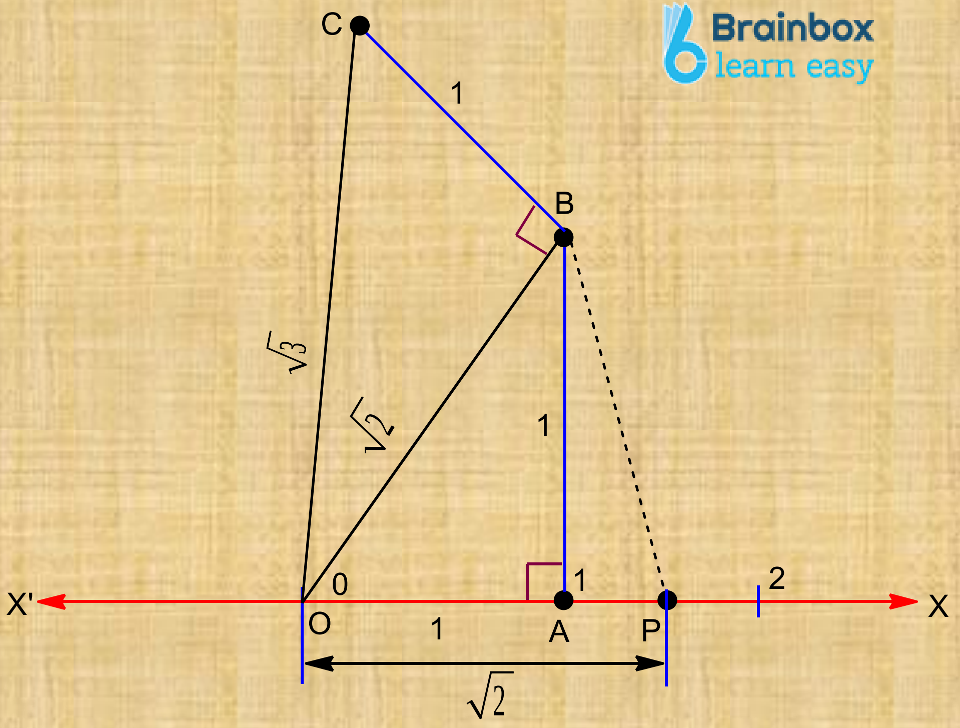
Step 5:

Now, draw BC = 1 unit, such that $BC \perp OB$.



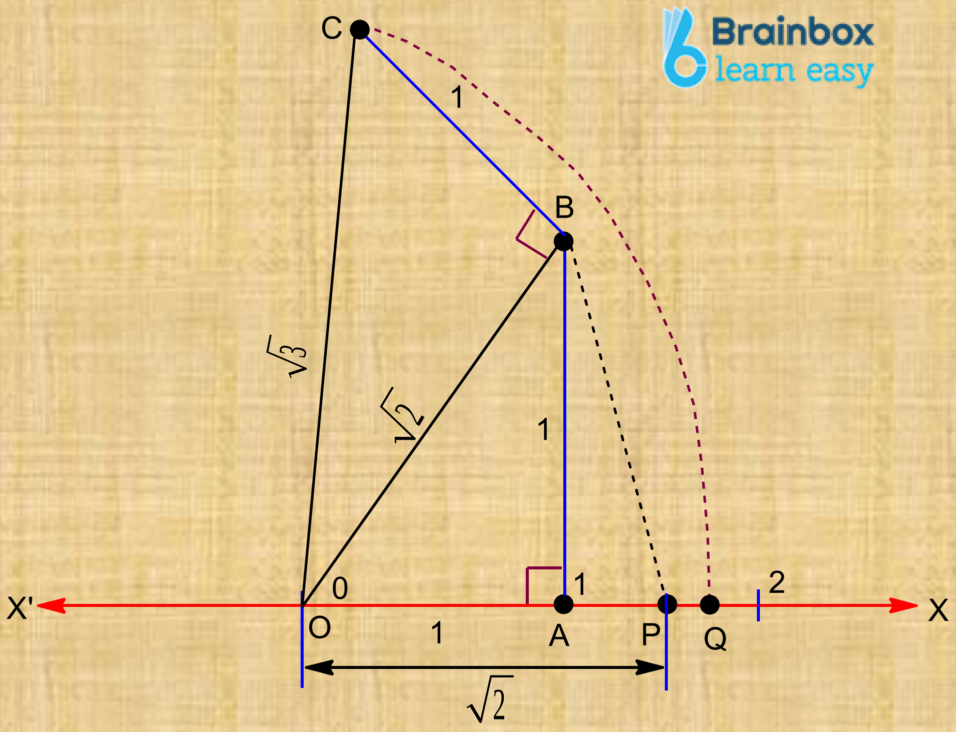
Step 6:

Join OC, we get OC = $\sqrt{OB^2 + BC^2} = \sqrt{(2)^2 + (1)^2} = \sqrt{2+1} = \sqrt{3}$ units



Step 7:

With ‘O’ as centre and radius OC = $\sqrt{3}$ units, draw an arc, which intersects OX at ‘Q’. Then OQ = OC = $\sqrt{1}$ unit.



Thus, the point ‘Q’ represents $\sqrt{3}$ on the number line.

Hence, the point ‘P’ and ‘Q’ represent the numbers $\sqrt{2}$ and $\sqrt{3}$ respectively.