CHAPTER 01

Number System

Method to determine rational numbers between two rational numbers:

As we know that, there are infinitely many (countless) rational numbers between two rational numbers, when we try to find out these rational numbers, we will follow the cases.

Case I:

Let 'a' and 'b' be two rational numbers such that a < b, then,

$q_1 = \frac{a+b}{2} \Rightarrow a < q_1 < b$ where q_1 is the rational number between 'a' and 'b'.

Now, $q_2 = \frac{a+q_1}{2} \Rightarrow a < q_2 < q_1 < b$, where q_2 is the rational number between a and q_1.

Now, $q_3 = \frac{q_1+b}{2} \Rightarrow a < q_2 < q_1 < q_3 < b$, where q_3 is the rational number between q_1 and b.

In this manner, we can find infinite rational numbers between two distinct rational numbers.

But this method is very time consuming.

Case II:

Let ‘a’ and ‘b’ be two rational numbers, such that b > a and we need to find ‘n’ rational numbers between ‘a’ and ‘b’ then, ‘n’ rational numbers lying between ‘a’ and ‘b’ are,

Method I:

(x + d), (x + 2d), (x + 3d)… (x + nd), where $d = \frac{b-a}{x+1}$.

Method II:

- If ‘a’ and ‘b’ are rational number with different denominator then make their denominator same by taking L.C.M.
- Now, multiply the numerator and denominator of ‘a’ and ‘b’ by (n + 1).

i.e. $a = \frac{a \times (n+1)}{(n+1)} = \frac{a(n+1)}{(n+1)}$ and $b = \frac{b \times (n+1)}{(n+1)} = \frac{b(n+1)}{(n+1)}$

- Let x_1, x_2, $x_3 \ldots x_n$ are ‘n’ rational numbers lying between ‘a’ and ‘b’ then,

$$x_1 = \left\{\frac{a(n+1)}{n+1} + 1\right\},\ x_2 = (x_1 + 1),\ x_3 = (x_2 + 1), \ldots\ x_n = (x_{n+1} + 1)$$

i.e. $a < x_1 < x_2 < x_3 \ldots < x_{n-1} < x_n < b$

Example:

Find the ‘5’ rational numbers between $\frac{1}{6}$ and $\frac{5}{24}$.

Sol.

Let $a = \frac{1}{6}$ or $\frac{4}{24}$, b = $\frac{5}{24}$ and n = 5

$\therefore$ a < b

Now, we have, $d = \frac{b-a}{n+1} = \frac{\left(\frac{5}{24} - \frac{1}{6}\right)}{(5+1)} = \frac{1}{14^4}$

Thus, 5 rational numbers between $\frac{1}{6}$ and $\frac{5}{24}$ are,

(a + d), (a + 2d), (a + 3d), (a + 4d) and (a + 5d)

$$\Rightarrow \left(\frac{1}{6} + \frac{1}{144}\right), \left(\frac{1}{6} + \frac{2}{144}\right), \left(\frac{1}{6} + \frac{3}{144}\right), \left(\frac{1}{6} + \frac{4}{144}\right) \text{ and } \left(\frac{1}{6} + \frac{5}{144}\right)$$

$$\Rightarrow \frac{25}{144}, \frac{26}{144}, \frac{27}{144}, \frac{28}{144}, \frac{29}{144}$$

$$\Rightarrow \frac{1}{6} < \frac{25}{144} < \frac{26}{144} < \frac{27}{144} < \frac{28}{144} < \frac{29}{144} < \frac{5}{24}$$

Example:

Find four rational number between – 4 and – 5.

Sol.

Here, – 4 > – 5 and n = 4.

Since, we need to find 4 rational numbers between – 4 and – 5.

So, multiply the numerator and denominator by (4 + 1) = 5 of – 4 and – 5.

i.e. $-4 = \frac{-4 \times 5}{5} = \frac{-20}{5}$ and $-5 = \frac{-5 \times 5}{5} = \frac{-25}{5}$

Thus, four rational numbers between – 4 and – 5 are,

$$\Rightarrow \frac{-21}{5}, \frac{-22}{5}, \frac{-23}{5}, \frac{-24}{5}$$

$$\Rightarrow \frac{-20}{5} > \frac{-21}{5} > \frac{-22}{5} > \frac{-23}{5} > \frac{-24}{5} > \frac{-25}{5}$$

$$\Rightarrow -4 > \frac{-21}{5} > \frac{-22}{5} > \frac{-23}{5} > \frac{-24}{5} > -5$$