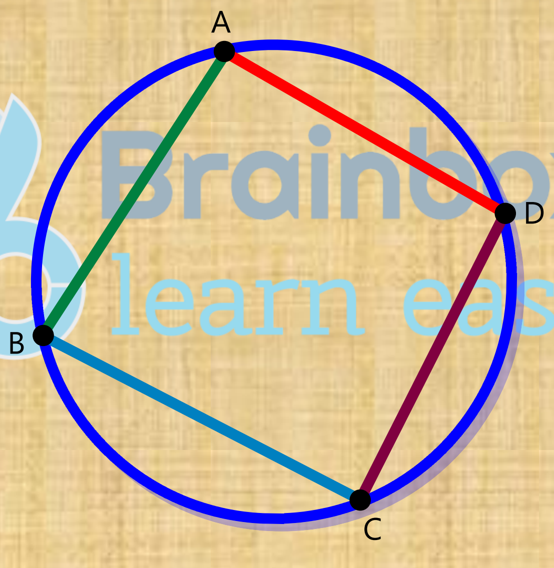
CHAPTER 10

Circles

Cyclic Quadrilaterals:

If four points are lie on the same circle is called a cyclic quadrilateral.

Let A, B, C, D are the four points lie on a circle, then ABCD is a cyclic quadrilateral.



Theorem 14:

Statement:

The sum of either pair of the opposite angels of a cyclic quadrilateral is 180°.

(OR)

The opposite angles of cyclic quadrilateral are supplementary.

Given:

A cyclic quadrilateral ABCD.

To prove:

$\angle A + \angle C = 180^\circ$ and $\angle B + \angle D = 180^\circ$.

Construction:

Join AC and BD.

Proof:

We have,

$\angle ACB = \angle ADB$ .... (1) (Angles in the same segment)

$\angle BAC = \angle BDC$ .... (2) (Angles in the same segment)

Adding (1) and (2), we get

$\angle ACB + \angle BAC = \angle ADB + \angle BDC$

$\Rightarrow \angle ACB + \angle BAC = \angle ADC$

$\Rightarrow \angle ACB + \angle BAC + \angle ABC = \angle ADC + \angle ABC$

(Adding $\angle ABC$ on both sides)

$\Rightarrow \angle ADC + \angle ABC = 180^\circ$

(Sum of the angles of ΔABC is 180^o)

$\Rightarrow \angle B + \angle D = 180^o$ .... (3)

Now,

$\angle A + \angle B + \angle C + \angle D = 360^o$

(Sum of the angles of a quadrilateral is 360^o)

$\Rightarrow \angle A + \angle C = 360^o - (\angle B + \angle D)$

$= 360^o - 180^o$ ($\because$ from (3))

$\Rightarrow \angle A + \angle C = 180^o$

Hence, $\angle A + \angle C = 180^o$ and $\angle B + \angle D = 180^o$.

Theorem 15: (Converse of theorem 14)

Statement:

If a pair of opposite angels of quadrilaterals is supplementary then the quadrilateral is cyclic.

Given:

A quadrilateral ABCD in which $\angle B + \angle D = 180^o$.

To prove:

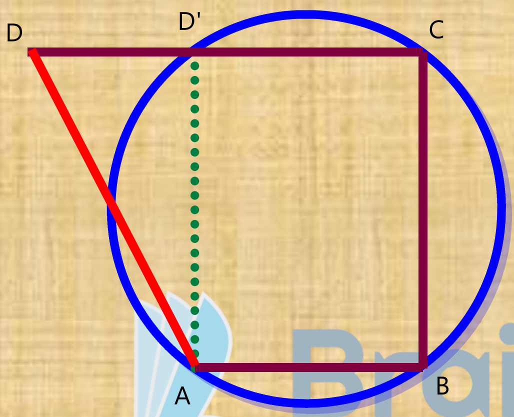
ABCD is a cyclic quadrilateral.

Construction:

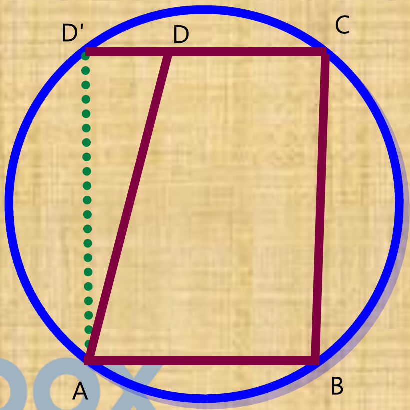
If possible, ABCD not a cyclic quadrilateral. Draw a circle, passing through three non – collinear points A, B, C.

Let this circle intersect CD in case (i) (or) CD produced in D'

at case (ii) Join D'A.



Case (i): If 'D' lies outside the circle



Case (ii): If 'D' lies inside the circle

Proof:

$\angle ABC + \angle ADC = 180^\circ$ (Given)

$\angle ABC + \angle AD'C = 180^\circ$

(Opposite angles of a cyclic quadrilateral)

$\therefore \angle ABC + \angle ADC = \angle ABC + \angle AD'C$

$\Rightarrow \angle ADC = \angle AD'C$

This is not possible, since an exterior angle of triangle can never be equal to its interior opposite angle.

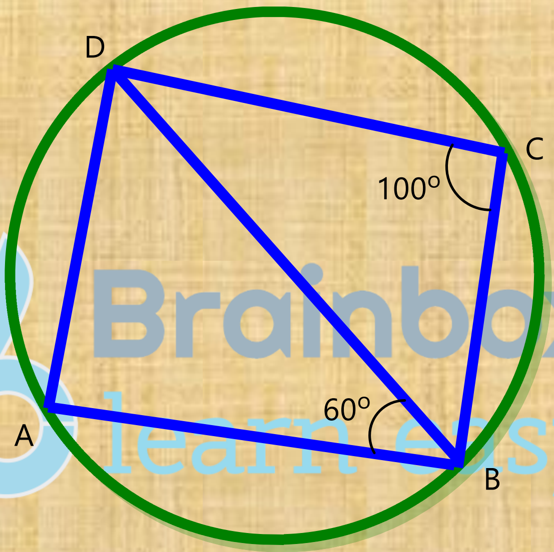
So, $\lfloor ADC = \lfloor AD'C$ is possible only when D' coincides with 'D'.

Hence, circle passing through A, B, C must pass through 'D' also.

Hence, ABCD is a cyclic quadrilateral.

Example:

In the given figure, ABCD is a cyclic quadrilateral. Find $\lfloor ADB$.



Sol.

We have,

$$\lfloor A + \lfloor C = 180^\circ$$

$$\Rightarrow \lfloor A + 100^\circ = 180^\circ$$

$$\Rightarrow \lfloor A = 180^\circ - 100^\circ = 80^\circ$$

In ΔADB, we have

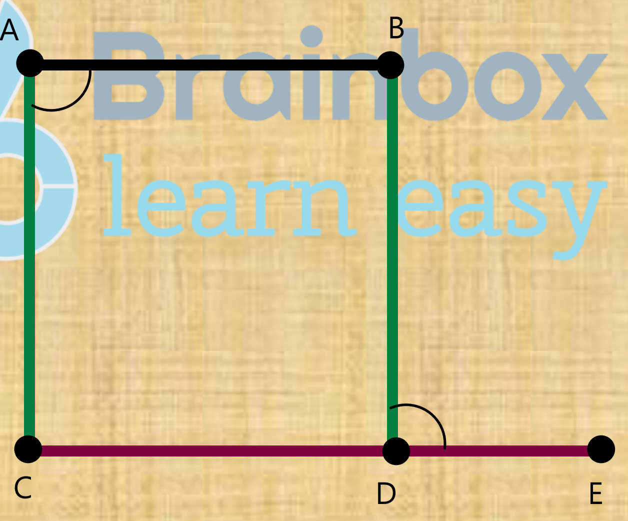
$$\Rightarrow \angle A = 80^\circ, \angle ABD = 60^\circ$$

$$\therefore \therefore \angle ADB = 180^\circ - \left(80^\circ + 60^\circ\right) = 40^\circ$$

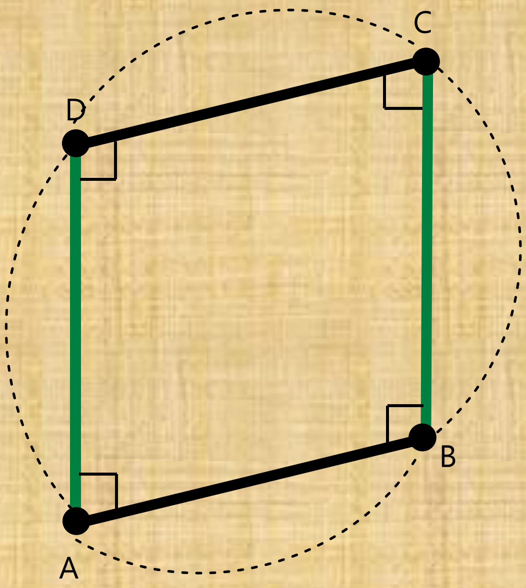
- If the sum of any pair of opposite angles of a quadrilateral is 180°, it is concyclic. If a side of a cyclic quadrilateral is produced, the exterior angle formed is equal to its opposite interior angle.

ABCD is a cyclic quadrilateral and here,

$\angle CAB = \angle BDE$

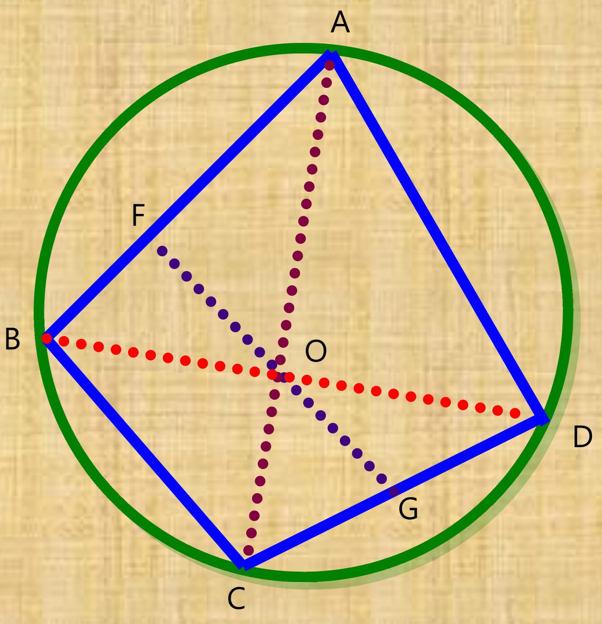


- If a parallelogram is cyclic, then it is a rectangle.



Brahmagupta theorem:

In a cyclic quadrilateral, in which the diagonals are perpendicular to each other, the perpendicular through the point of intersection of the diagonals to one side bisects the opposite side and conversely.



Note:

The polygon inscribed in a circle is called a concyclic polygon.