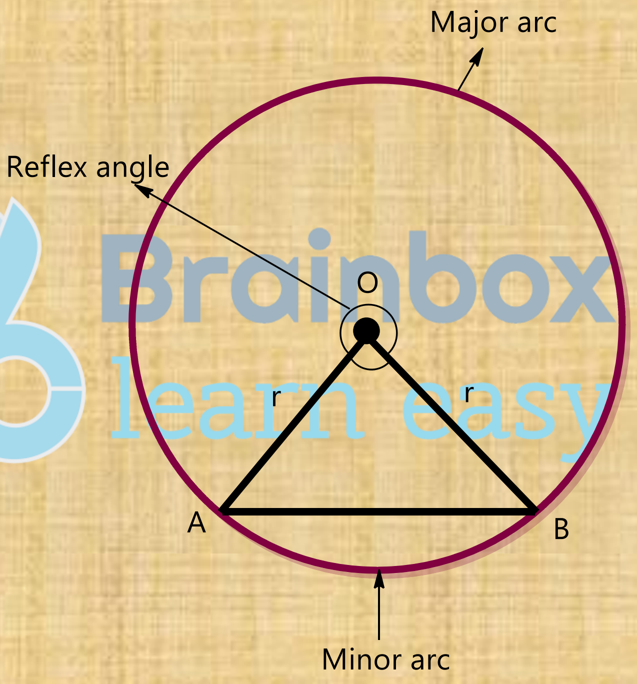
## CHAPTER 10

# Circles

**Angle subtended by an arc of a circle:**

The angle formed by the two radii at the ends of an arc of the circle, at the centre of the circle is called the angle subtended by the arc.

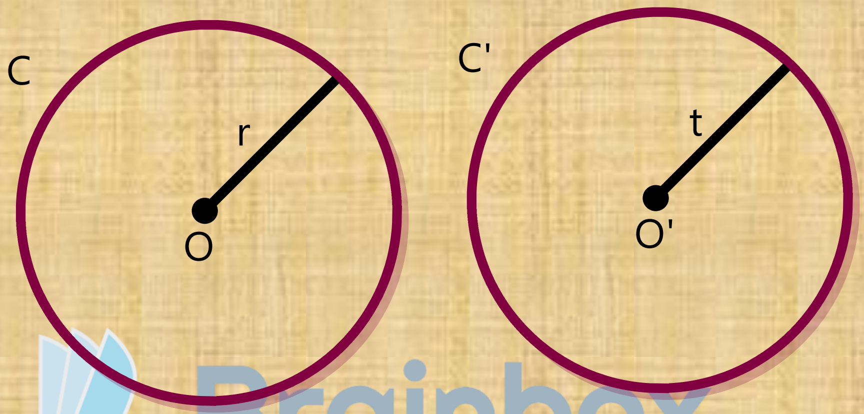



**Note:**

- $\angle AOB$ is the angle subtended by the minor arc AB of a circle with centre 'O'.
- The angle subtended by the major arc AB at 'O' is reflex $\angle AOB$.

**Congruent circles:**

Two circles are said to be congruent if and only if either of them can be superimposed on the other so as to cover it exactly. Two circles are said to be congruent if and only if their radii are equal.

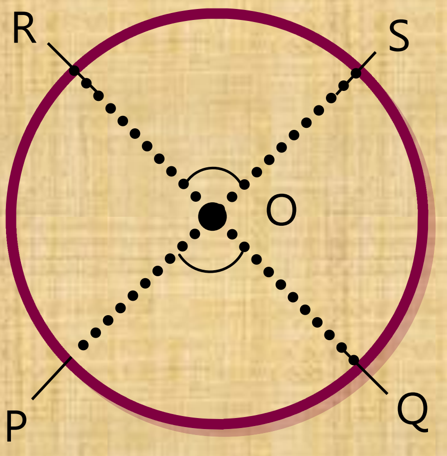



If r = t



**Congruent arcs:**

Two arcs of a circle (or of congruent circles) are said to be congruent if they have the same degree measure.



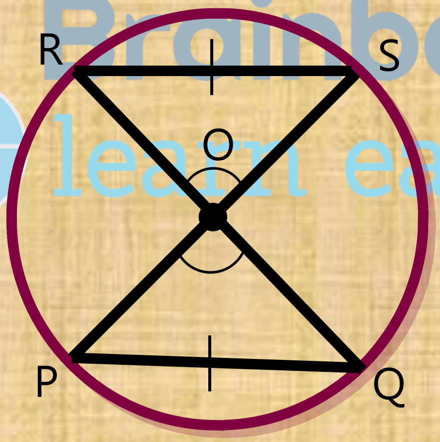
If two arcs PQ and RS are congruent arcs of a circle with centre 'O' and radius 'r'. We write PQ = RS.

Thus, $PQ \cong RS \Leftrightarrow m(PQ) = m(RS) \Rightarrow \angle POQ = \angle ROS$

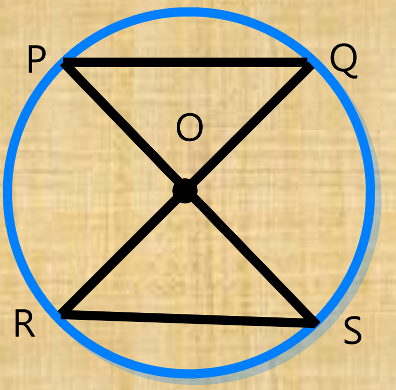
i.e. Equal arcs (or Congruent arcs) of a circle subtend equal angles at the centre.

**Note:**

- If two arcs of a circle (or of Congruent circles) are congruent, then corresponding chords are equal.

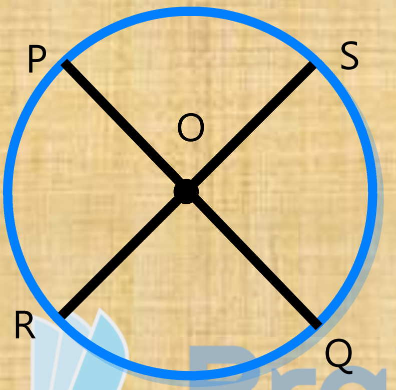



- If two chords of a circle (or of Congruent circles) are equal, then their corresponding arcs (Minor, Major or Semicircular) are congruent.




$\overset{\frown}{QP}$, $\overset{\frown}{SR}$ are major arcs
and $\overset{\frown}{QP} \cong \overset{\frown}{SR}$

$\overset{\frown}{PQ}$, $\overset{\frown}{RS}$ are minor arcs
$\overset{\frown}{PQ} \cong \overset{\frown}{RS}$

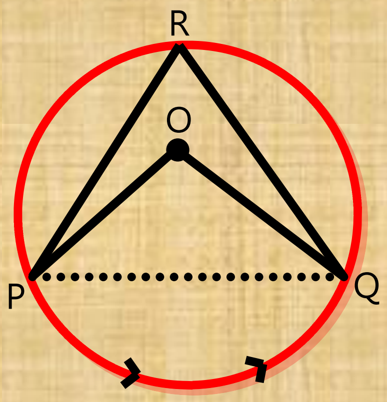



$\overset{\frown}{PQ}$, $\overset{\frown}{RS}$ are semicircles
with the same radii
$\overset{\frown}{PQ} \cong \overset{\frown}{RS}$

**Theorem 10:**

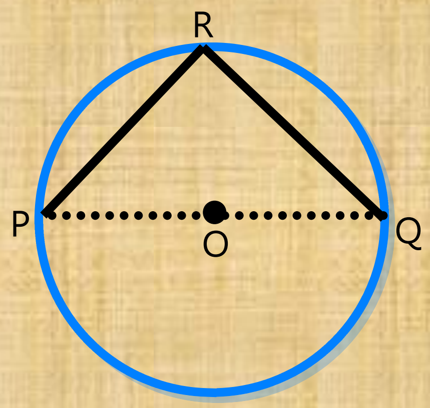
**Statement:**

The angle subtended by an arc of a circle at the centre is double the angle subtended by it at any point on the remaining part of the circle.



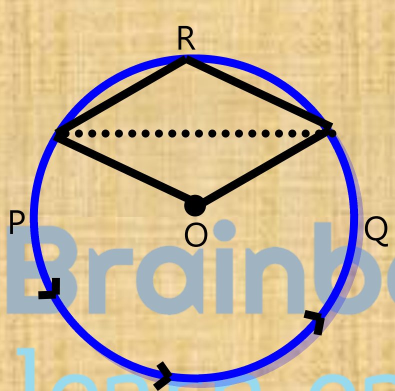

When $\widehat{PQ}$ is a minor arc

$\angle POQ = 2\angle PRQ$




When PQ is a diamter

$\angle POQ = 2\angle PRQ$




When $\widehat{PQ}$ is a major arc

Reflex $\angle POQ = 2\angle PRQ$

**Angle in a semicircle:**

**Theorem 11:**

**Statement:**

The angle in a semicircle is a right angle.

**Given:**

PQ is a diameter of a circle with centre ‘O’ and $\angle PRQ$ is an angle in semicircle.

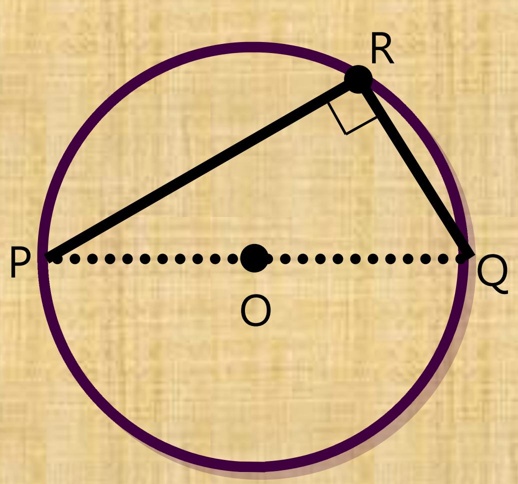
**To prove:**

$\angle PRQ = 90^\circ$



**Proof:**

We know that the angle subtended by an arc of a circle at its centre is twice the angle formed by same arc at a point on the circle. So, we have



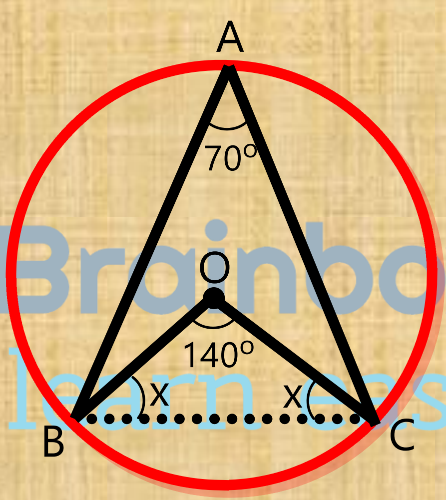
$$\angle POQ = 2\angle PRQ$$

$$\Rightarrow 180^\circ = 2\angle PRQ \qquad (\because \text{ POQ is a straight line})$$

$$\Rightarrow \angle PRQ = \frac{180^\circ}{2} = 90^\circ$$

**Example:**

Find the value of 'x' in the given figure.




**Sol.**

Given,

$$\angle BAC = 70^\circ$$

$$\angle BOC = 2\angle BAC$$

($\because$ Angle subtended by an arc at the centre is double the angle subtended by it at any point on the remaining part of the circle)

$\therefore \angle BOC = 2 \times 70 = 140^\circ$

Now, In $\Delta BOC$, OB = OC (Radii of same circle)

$\therefore \angle OBC = \angle OCB = x$

($\because$ Angles opposite to equal sides of a triangle are equal)

Also, $\angle OBC + \angle OCB + \angle BOC = 180^\circ$

(By angle sum property of a triangle)

$x + x + 140^\circ = 180^\circ$

$\Rightarrow 2x = 180^\circ - 140^\circ = 40^\circ$

$\Rightarrow \quad x = \dfrac{40^\circ}{2}$

$\therefore \; x = 20^\circ$

**Angles in the same segment:**

**Theorem 12:**

Angles in the same segment of a circle are equal.

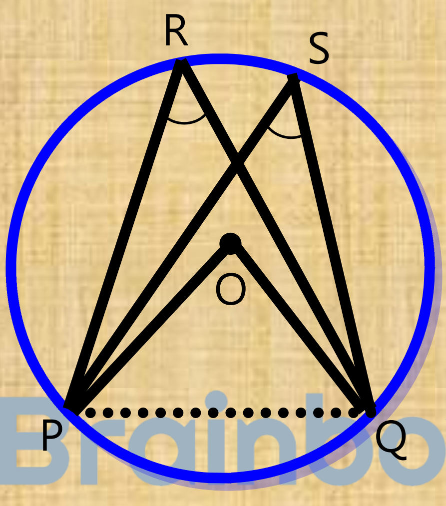
Consider the figure, where PQ is an arc, R and S are any two points on the remaining part of the circle.

Suppose, we join points 'P' and 'Q', form a chord PQ.

Then $\angle PRQ$ is also called the angle formed in the segment PRQP.

Here, $\angle PRQ = \angle PSQ$

i.e. Angles in the same segment are equal.




**Theorem 13: (Converse of theorem 12)**

**Statement:**

If a line segment joining two points subtends equal angles at two other points lying on the same side of the line containing the line segment, the four points lie on a circle.

i.e., They are concyclic.