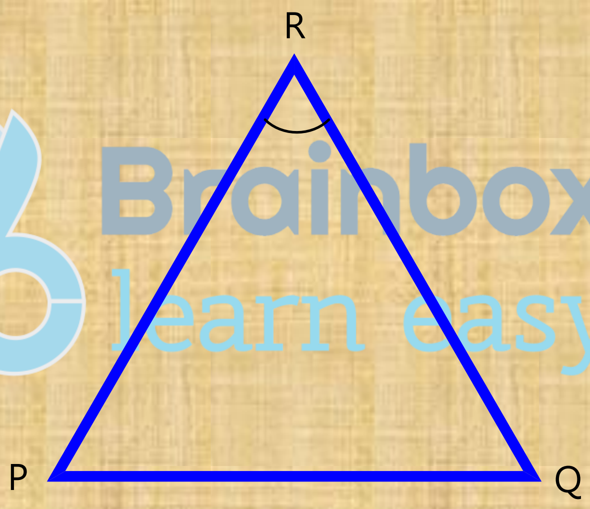
CHAPTER 10

Circles

Angle subtended by a chord at a point:

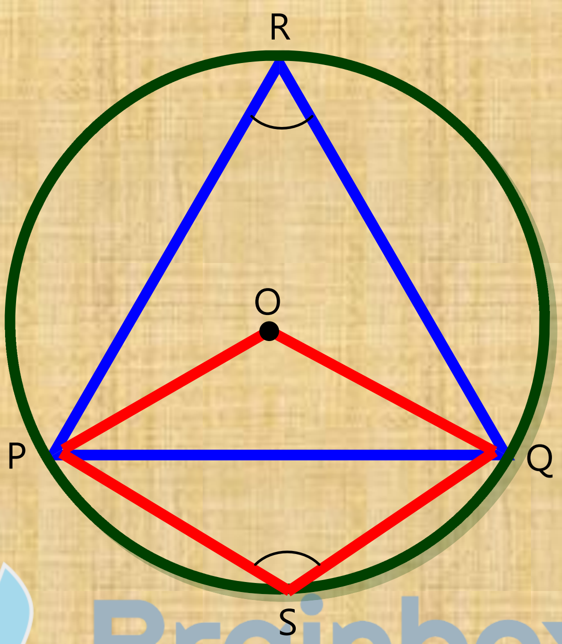
Consider three non-collinear point on a plane. Draw a line segment PQ, then by joining PR and QR. We get $\angle PRQ$, which is called the angle subtended by the line segment PQ at the point 'R'.



From the figure:

- We can say $\angle POQ$ is the angle subtended by the chord PQ at the centre 'O'.
- $\angle PRQ$ is the angle subtended by the chord PQ at the point 'R' on the major arc PQ.

- $\angle PSQ$ is the angle subtended by the chord PQ at the point 'S' on the minor PQ.



Example:

Find 'x' and angle subtended by chord PQ at the centre 'O'.

Sol.

Given,

$\angle QOR = 60^o$

$\angle QPO = ?$

$\angle POQ = ?$

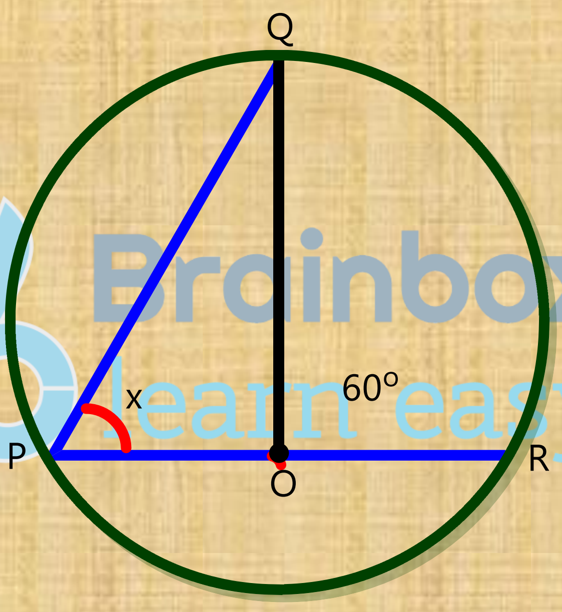
We know that,

$\angle POQ + \angle QOR = 180^\circ$ (∵ Linear)

$\Rightarrow \angle POQ + 60^\circ = 180^\circ$

$\Rightarrow \angle POQ = 180^\circ - 60^\circ$

$\therefore \angle POQ = 120^\circ$



Here, OQ = OP (radius)

$\Rightarrow \angle OQP = \angle OPQ$

i.e. $\angle Q = \angle P$ (Angles opposite to equal sides of a triangle are equal)

$\Rightarrow \angle Q = \angle x$

Now, In ΔPOQ

$\angle P + \angle O + \angle Q = 180^\circ$ ($\because$ Angles sum property of a triangle)

$\Rightarrow x + 120^\circ + x = 180^\circ$

$\Rightarrow 2x = 180^\circ - 120^\circ = 60^\circ$

$\Rightarrow x = \dfrac{60^\circ}{2} = 30^\circ$

Hence, $\angle QPO = 30^\circ$

Theorem (1):

Statement:

Equal chords of a circle subtend equal angles at the centre.

Given:

A circle with centre 'O' in which Chord AB = Chord CD

To Prove:

$\angle AOB = \angle COD$

Proof:

In ΔAOB and ΔCOD, we have

OA = OC (Radii of circle)

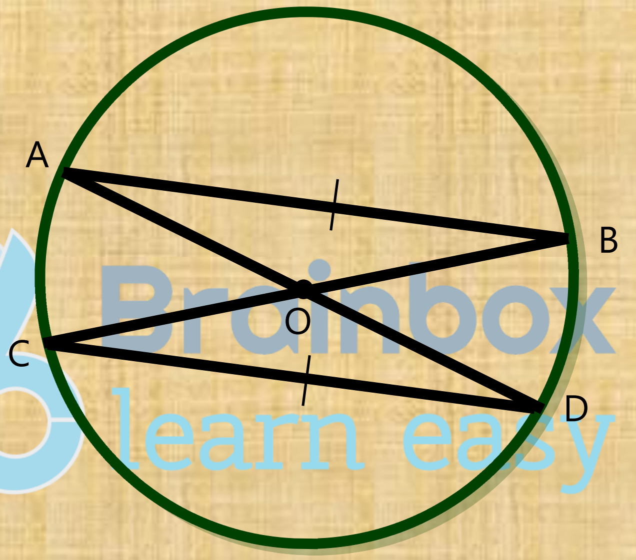
OB = OD (Radii of circle)

AB = CD (Given)

$\therefore\ \Delta AOB \cong \Delta COD$ (By SSS Congruence criteria)

Hence, $\angle AOB = \angle COD$ (C.P.C.T)

Where C.P.C.T = Corresponding Parts of Congruent Triangles



Theorem 2: (Converse of theorem 1)

Statement:

If the angles subtended by the chords of a circle at the centre are equal, then the chords are equal.

Given:

A circle with centre 'O' in which AB and CD are the chords such that $\angle AOB = \angle COD$.

To prove:

AB = CD

Proof:

In ΔAOB and ΔCOD, we have

OA = OC (Radii of circle)

OB = OD (Radii of circle)

$\angle AOB = \angle COD$ (Given)

$\therefore \Delta AOB \cong \Delta COD$ (By S.A.S Congruence criteria)

Hence, AB = CD (C.P.C.T)