CHAPTER 09

Areas of Parallelogram and Triangles

Example 1:

D, E and F are respectively the midpoints of the sides BC, CA and AB of ΔABC. Show that

(i) BDEF is a parallelogram.

(ii) Area (ΔDEF) = $\frac{1}{4}$ Area (ΔABC)

(iii) Area (Parallelogram BDEF) = $\frac{1}{2}$ Area (ΔABC)

Sol.

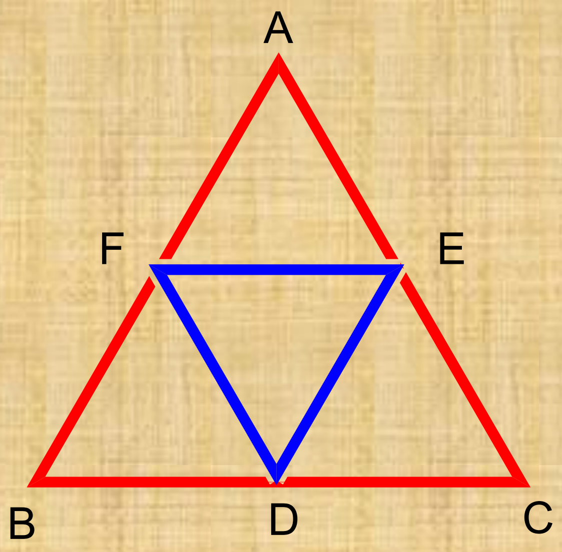
We have ΔABC such that D, E and F are the midpoints of BC, CA and AB respectively.

(i) To prove that, BDEF is a parallelogram.

In ΔABC, E and F are the midpoints of AC and AB respectively.

$\therefore EF \parallel BC$ (By midpoint theorem)

Also, $EF = \frac{1}{2} BC$



$\Rightarrow EF = BD$ [Where D is the midpoint of BC]

Since, BDEF is a quadrilateral whose one pair of opposite sides is parallel sides is parallel and of equal lengths.

$\therefore$ BDEF is a parallelogram.

(ii) We have proved that BDEF is a parallelogram.

Similarly, DCEF is a parallelogram and DEAF is a parallelogram.

Now, parallelogram BDEF and parallelogram DCEF, on the same base EF and between the same parallels BC and EF.

$\therefore$ Area (Parallelogram BDEF) = Area (Parallelogram DCEF)

$\Rightarrow \frac{1}{2}$ Area (Parallelogram BDEF) = $\frac{1}{2}$ Area (Parallelogram DCEF)

$\Rightarrow$ Area (ΔBDF) = Area (ΔCDE)… (1)

Diagonal of a parallelogram divides it into two triangles of equal area.

Similarly, Area (ΔCDE) = Area (ΔDEF)……… (2)

And

Area (ΔAEF) = Area (ΔDEF)… (3)

From (1), (2) and (3), we have

Area (ΔAEF) = Area (ΔFBD) = Area (ΔDEF) = Area (ΔCDE)

$\therefore$ Area (ΔABC) = Area (ΔAEF) + Area (ΔFBD) + Area (ΔDEF) + Area (ΔCDE)

Area (ΔABC) = 4 x Area (ΔDEF)

Area (ΔDEF) = $\frac{1}{4}$ Area (ΔABC)

(iii) We have

Area (Parallelogram BDEF) = Area (ΔBDF) + Area (ΔDEF)

= Area (ΔDEF) + Area (ΔDEF)

[$\because$ Area (ΔDEF) = Area (ΔBDF)]

= 2 Area (ΔDEF)

= 2 [$\frac{1}{4}$ Area (ΔABC)]

= $\frac{1}{2}$ Area (ΔABC)

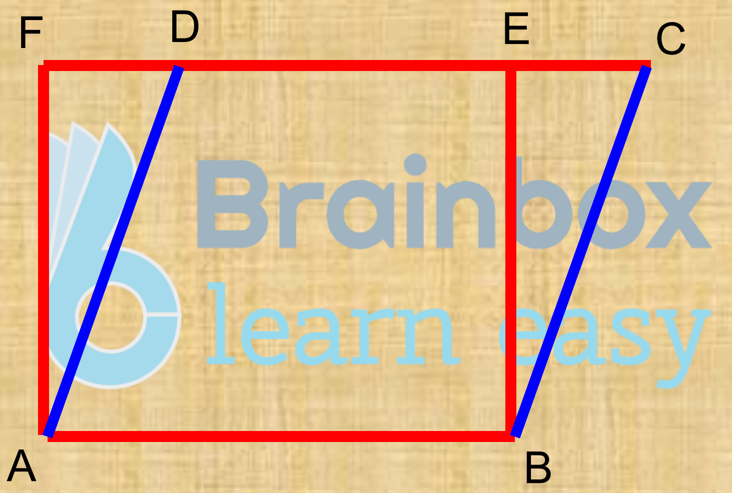
$\therefore$ Area (Parallelogram BDEF) = $\frac{1}{2}$ Area (ΔABC)

Example 2:

Parallelogram ABCD and Rectangle ABEF are on the same base AB and have equal areas. Show that the perimeter of the parallelogram is greater than that of the rectangle.

Sol.

We have a parallelogram ABCD and rectangle ABEF.



Such that, Area (Parallelogram ABCD) = Area (Rectangle ABEF)

AB = CD (Opposite sides of a parallelograms)

AB = EF (Opposite sides of a rectangle)

$\Rightarrow$CD = EF

$\Rightarrow$AB + CD = AB + EF …………….. (1)

$\therefore$BE < BC and AF < AD (In a right triangle hyp. is the longest side)

$\Rightarrow$(BC + AD) > (BE + AF) …………….. (2)

From (1) & (2), we have

(AB + CD) + (BC + AD) > (AB + EF) + (BE + AF)

$\Rightarrow$(AB + BC + CD + DA) > (AB + BE + EF + FA)

$\Rightarrow$Perimeter of parallelogram ABCD > Perimeter of rectangle ABEF.