Brainbox learn easy

Chapter 02

POLYNOMIALS

Algebraic Identities:

i) $(a+b)^2 = a^2 + 2ab + b^2$

ii) $(a-b)^2 = a^2 - 2ab + b^2$

iii) $(a+b)(a-b) = a^2 - b^2$

iv) $(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$ (or)

$(a+b)^3 = a^3 + 3ab(a+b) + b^3$

v) $(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$ (or)

$(a-b)^3 = a^3 - 3ab(a-b) - b^3$

vi) $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$

vii) $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

viii) $(a+b)(c+d) = ac + ad + bc + bd$

ix) $(x+a)(x+b) = x^2 + x(a+b) + ab$

x) $(a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

xi) $(a+b)^2 + (a-b)^2 = 2(a^2 + b^2)$

xii) $(a+b)^2 - (a-b)^2 = 4ab$

xiii) $ab = \left(\frac{a+b}{2}\right)^2 - \left(\frac{a-b}{2}\right)^2$

xiv) $a^k + b^k = (a^{k-1} + b^{k-1})(a+b) - (a^{k-2} + b^{k-2})ab$

xv) $a^3 + b^3 + c^3 - 3abc = (a+b+c)(a^2 + b^2 + c^2 - ab - bc - ca)$

Special Identities:

1) If $a+b+c=0$ then

- $a^2+b^2+c^2=-2(ab+bc+ca)$
- $a^3+b^3+c^3=3abc$
- $a^4+b^4+c^4=2(a^2b^2+b^2c^2+c^2a^2)=\frac{1}{2}(a^2+b^2+c^2)^2$
- $a^5+b^5+c^5=-5abc(ab+bc+ca)=\frac{5}{2}abc(a^2+b^2+c^2)$
  $=\frac{5}{6}(a^2+b^2+c^2)(a^3+b^3+c^3)$
- $a^7+b^7+c^7=7abc(ab+bc+ca)^2=\frac{7}{12}(a^2+b^2+c^2)(a^3+b^3+c^3)$
  $=\frac{7}{6}(a^4+b^4+c^4)(a^3+b^3+c^3)$

2) If $\frac{1}{a}+\frac{1}{b}+\frac{1}{c}=\frac{1}{a+b+c}$ then

$$\frac{1}{a^{2n+1}}+\frac{1}{b^{2n+1}}+\frac{1}{c^{2n+1}}=\frac{1}{(a+b+c)^{2n+1}}.$$