CHAPTER 01

Rational Numbers

Laws of rational numbers:

Properties:

We will discuss about reciprocal of a number and additive inverse of a number.

Closure property:

As per addition, sum of two rational numbers is always a rational number. If you have any two rational numbers. Let us say a and b any two rational numbers, sum of a and b is always a rational number c.

Example:

$$\left(\frac{-5}{7}\right)+\frac{3}{8}=\frac{-40+21}{56}=\frac{-19}{56}$$ is a rational number.

We can say that rational numbers are closed under addition.

Will the difference of two rational numbers again a rational number?

Example:

$$\left(\frac{-5}{7}\right)-\frac{2}{3}=\frac{-15-14}{21}=\frac{-29}{21}$$ is a rational number.

From the above example, we can say that rational numbers are closed under subtraction.

We can conclude that for any two rational numbers a and b, a – b is also a rational number.

Product of two rational numbers:

$$\left(\frac{-2}{3}\right)\times\frac{4}{5}=\frac{-8}{15}$$

We can say that, rational numbers are closed under multiplication.

We can conclude that for any two rational numbers a and b, a x b is also a rational number.

Can we say that, rational numbers closed under division?

Example:

$\frac{2}{7}\div\left(\frac{-5}{3}\right)=\frac{2}{7}\times\frac{3}{-5}=\frac{6}{-35}$ is a rational number.

Let us observe one more example,

For any rational number 2, $2\div 0$ is not defined. So, rational numbers are not closed under division.

So, we can conclude that, if we exclude zero then the collection of all other rational numbers is closed under division.

Commutative property:

Example:

$$\frac{-2}{3}+\frac{5}{7}=\frac{-14+15}{21}=\frac{1}{21} \text{ and}$$

$$\frac{5}{7}+\left(\frac{-2}{3}\right)=\frac{15-14}{21}=\frac{1}{21}$$

Above in both cases, we got same answers.

$$\therefore \quad \frac{-2}{3}+\frac{5}{7}=\frac{5}{7}+\left(\frac{-2}{3}\right)$$

We can say that, two rational numbers can be added in any order.

So, we can conclude that addition is commutative for rational numbers.

i.e., $\boxed{a+b=b+a}$

Subtraction of two rational numbers:

Example:

$$\frac{2}{3}-\frac{5}{4}=\frac{8-15}{12}=\frac{-7}{12} \text{ and}$$

$$\frac{5}{4}-\frac{2}{3}=\frac{15-8}{12}=\frac{7}{12}$$

Above in both cases, we got different answers.

Therefore, $\frac{2}{3}-\frac{5}{4}\neq\frac{5}{4}-\frac{2}{3}$

So, subtraction is not commutative for rational numbers.

We can conclude that, for any two rational numbers a and b,

$$\boxed{a-b\neq b-a}$$

Multiplication of rational numbers:

Example:

$\frac{-2}{3}\times\frac{5}{4}=\frac{-10}{12}$ and

$\frac{5}{4}\times\left(\frac{-2}{3}\right)=\frac{-10}{12}$

$$\therefore\ \boxed{\frac{-2}{3}\times\frac{5}{4}=\frac{5}{4}\times\left(\frac{-2}{3}\right)}$$

We can find that, multiplication of any two rational numbers is commutative for rational numbers.

In general, we can conclude that, a x b = b x a, for any two rational numbers a and b.

Division of any two rational numbers:

Let us observe that,

$\left(\frac{-5}{4}\right)\div\frac{3}{7}=\frac{-5}{4}\times\frac{7}{3}=\frac{-35}{12}$ and

$$\frac{3}{7} \div \left(\frac{-5}{4}\right) = \frac{3}{7} \times \frac{4}{-5} = \frac{12}{-35}$$

$\therefore \left(\frac{-5}{4}\right) \div \frac{3}{7} \neq \frac{3}{7} \div \left(\frac{-5}{4}\right)$, we can say that, division is not commutative under division of any two rational numbers.

So, $a \div b \neq b \div a$.