

Chapter 09

ALGEBRAIC EXPRESSION & IDENTITIES

Example- 4:

If $x + \frac{1}{x} = 3$, find the value of $x^4 + \frac{1}{x^4}$.

Solution:

We have,

$$x + \frac{1}{x} = 3$$

Squaring on both sides, we get

$$\left(x + \frac{1}{x}\right)^2 = (3)^2$$

$$x^2 + 2\left(\cancel{x}\right)\left(\frac{1}{\cancel{x}}\right) + \left(\frac{1}{\cancel{x}}\right)^2 = 9 \quad \left(\because (a+b)^2 = a^2 + 2ab + b^2\right)$$

$$x^2 + \frac{1}{x^2} = 9 - 2 = 7$$

Again squaring on both sides,

$$\left(x^2 + \frac{1}{x^2}\right)^2 = (7)^2$$

$$\left(x^2\right)^2 + 2\left(\cancel{x^2}\right)\left(\frac{1}{\cancel{x^2}}\right) + \left(\frac{1}{\cancel{x^2}}\right)^2 = 49 \quad \left(\because (a+b)^2 = a^2 + 2ab + b^2\right)$$

$$x^4 + \frac{1}{x^4} = 49 - 2$$

$$\therefore \boxed{x^4 + \frac{1}{x^4} = 47}$$