

CHAPTER 09

Tangents and Secants to a Circle

Line and Circle:

Consider a circle with centre 'O' and radius 'r' also consider a line 'l' in a plane. We have three possibilities namely.

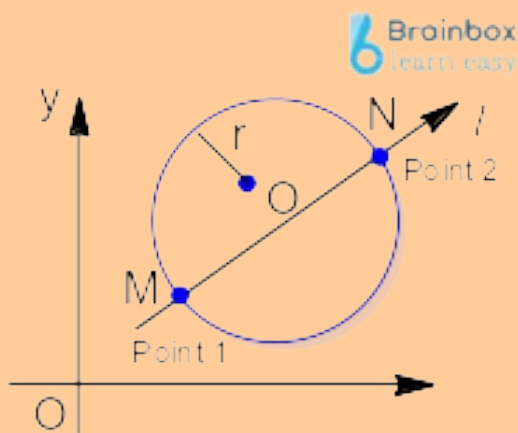
Case (i):

The line meets the circle in two distinct points.

Secant:

A line, which intersects the circle at two distinct points, is called a 'Secant'.

In the given figure, the line 'l' cuts the circle at the points M and N. So, 'l' is the secant of the circle.



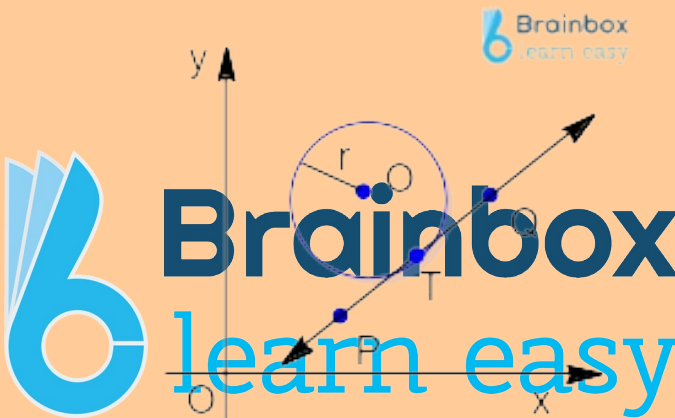
Case (ii):

The line meets the circle in one and only one point.

Tangent:

A line, which touches a circle at one point, only is called a 'Tangent' to the circle at that point.

In the given figure, PTQ is a tangent to the circle at point 'T'.



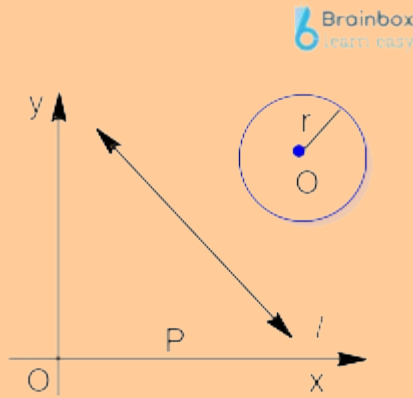
Note:

- The point at which the tangent touches the circle is called the 'Point of contact'.
- One and only one tangent can be drawn to a circle at a point.
- All the points on a tangent to a circle, the point of contact is nearest to the centre of the circle.

Case (iii):

The line ' l ' does not meet the circle.

i.e. ' l ' and the circle have no common points.



Theorem 1:

Statement:

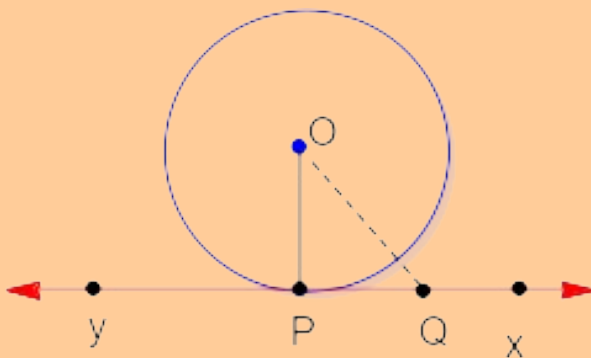
The tangent at any point of a circle is perpendicular to the radius through the point of contact.

Given:

A circle with centre 'O' and a tangent xy to the circle at a point 'P'.

To prove:

$$OP \perp xy$$



Proof:

- Take a point 'Q' on xy other than 'P' and join OQ.

- The point 'Q' must lie outside the circle.
- Therefore, OQ is longer than the radius OP of the circle.

$$OQ > OP$$

- This happens for every point on the line xy except the point 'P'.
- OP is the shortest of all the distances of the point 'O' to the points on xy. Hence, OP is perpendicular to xy.

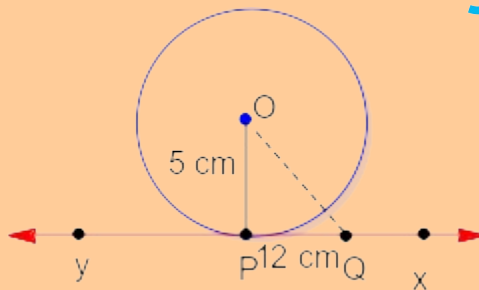
Example:

A tangent PQ at a point 'P' of a circle radius 5 cm meets a line through the centre 'O' at a point 'Q'. So that $PQ = 12\text{cm}$. Find the length of OQ.

Sol.

Since, tangent at any point of circle is perpendicular to the radius through the point of contact.

$\therefore OP \perp PQ \Rightarrow \triangle OPQ$ is right angle triangle.



In right angle, OPQ

$$\begin{aligned} OQ^2 &= OP^2 + PQ^2 \\ &= (5)^2 + (12)^2 \\ &= 25 + 144 \end{aligned}$$

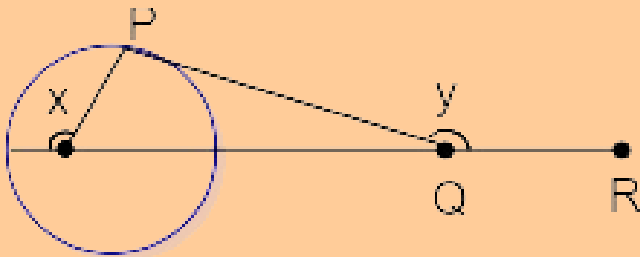
$$OQ^2 = 169$$

$$OQ = \sqrt{169}$$

$$\therefore \boxed{OQ = 13 \text{ cm}}$$

Example:

From the given figure, find the value of $x + y$.



Sol.

Tangent at any point of a circle is perpendicular to the radius through the point of contact.

$$\angle OPQ = \angle OQP = 90^\circ$$

In $\triangle OPQ$,

$$\angle PQR = \angle OPQ + \angle POQ \quad (\text{By exterior angle property})$$

$$\Rightarrow y = 90^\circ + (180^\circ - x)$$

$$\Rightarrow y = 270^\circ - x$$

$$\therefore \boxed{x + y = 270^\circ}$$