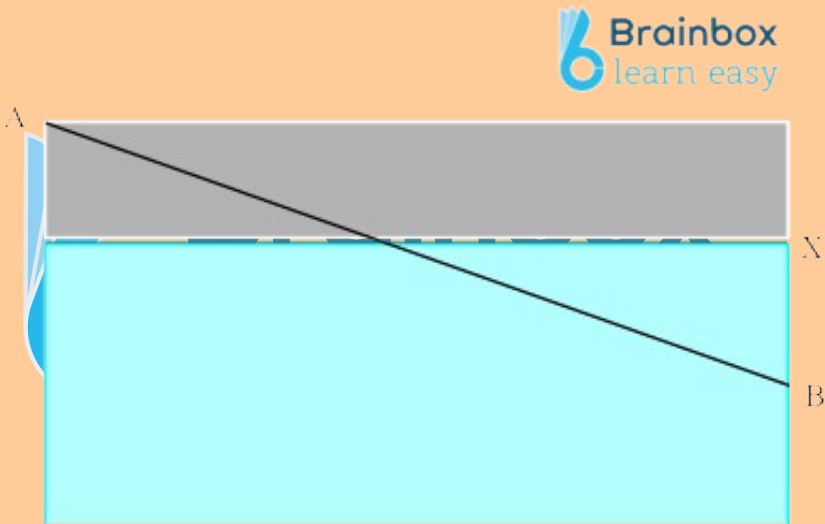


Refraction of Light at Plane Surfaces

Derivation of Snell's law:

Imagine that a person has fallen out of boat and is screaming for help in the water at point as shown in figure.

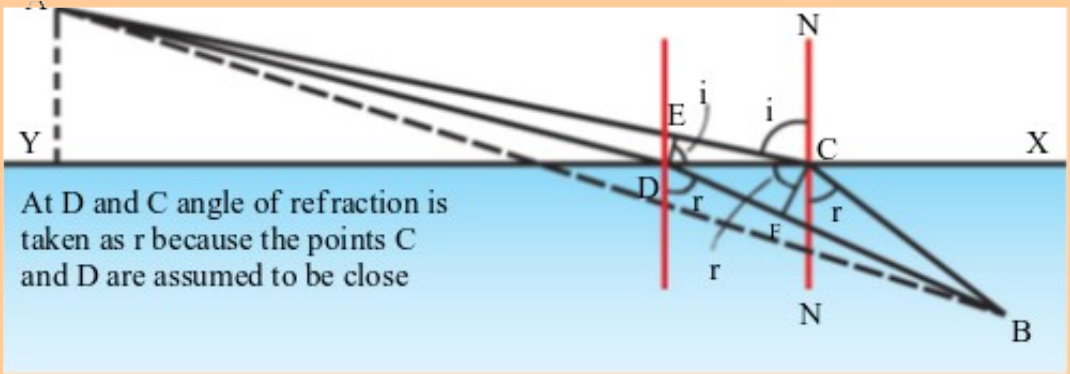


The line marked through point 'X' is the shoreline. Let us assume that we are at a point 'A' on the shore and we saw the accident. In order to save the person we need to travel a certain distance on land and a certain distance in water.

We know that, we can run faster on land than we can swim in water.

- What do we do to save the person?
- Which path enables us to save the person in the shortest possible time?
- Do we go in a straight line?

By careful thought, we would realize that it would be advantageous to travel a greater distance on the land in order to decrease the distance in water because we go much slower in water. For whatever speeds on land and in water, the final path that one has to follow to reach the person is ACB, and that this path takes the shortest time of all the possible paths.



If we take any other route, it will be longer.

Where 'C', the point on shoreline, corresponds to the shortest of all possible times. Let us consider a point 'D' on shoreline, which is very close to point 'C' such that there is essentially no change in time between path ACB and ADB.

Let us try to calculate how long it would take to go from A to B by the two paths one through point D and another through point C. First, look at the paths on the land as shown in fig. If we draw a perpendicular DE; between two paths at D, we see that the path (AD) on land is shortened by the amount EC. On the other hand, in the water, by drawing a corresponding perpendicular CF we find that we have to go the extra distance DF in water. In other words, we gain a time that is equal to go through distance EC on land but we lose the time that is equal to go extra distance DF in water. These times must be equal since we assumed there no change in time between the two paths.

Let the time taken to travel from E to C and D to F be Δt and v_1 and v_2 be the speeds of running and swimming. From fig. we get,

$$EC = v_1 \Delta t \text{ and } DF = v_2 \Delta t$$

$$\Rightarrow \frac{EC}{DF} = \frac{v_1}{v_2}$$

Let i and r be the angles measured between the path ACB and normal NN, perpendicular to shore line X.

$$\sin i = \frac{EC}{DC} \quad \text{and} \quad \sin r = \frac{DF}{DC}$$

When we divide these 2 equations, we have,

$$\frac{\sin i}{\sin r} = \frac{EC}{DF}$$

As we have already seen $\frac{\sin i}{\sin r} = \frac{v_1}{v_2}$.

Thus to save the person, one should take such a path to satisfy the above equation. We used the principle of least time to derive the above result. Hence we can apply the same for the light ray also. We get,

$$\frac{\sin i}{\sin r} = \frac{v_1}{v_2} = \frac{n_1}{n_2}$$

$$n_1 \sin i = n_2 \sin r$$

This is called Snell's law.

From our discussions so far we can come to some conclusions, which we call laws of refraction

1. Incident ray, the refracted ray and the normal to interface of two transparent media at the point of incidence all lie in the same plane.
2. During refraction, light follows Snell's law.